Isolating the Effect of Day-Count Conventions on the Market Value of Interest Rate Swaps

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Abstract

Day-count conventions are a ubiquitous but often overlooked aspect of interest-bearing investments. While many market traded securities have adopted fixed or standard conventions, over-the-counter agreements such as interest rate swaps can and do use a wide variety of conventions, and many investors may not be aware of the effects of this choice on their future cash flows. Here, we show that the choice of day-count convention can have a surprisingly large effect on the market value of swap agreements. We highlight the importance of matching day-count conventions on obligations and accompanying swap agreements, and demonstrate various factors which influence the magnitude of day-count convention effects. As interest rate swaps are very common amongst municipal and other institutional investors, we urge investors to thoroughly understand these and other ‘fine print’ terms in any potential agreements. In particular, we highlight the ability of financial intermediaries to effectively increase their fees substantially through their choice of day-count conventions.

1 Introduction

We have different day-count conventions today for the same reason we have different driving conventions in the US and the UK – history. Historically some securities have had their interest payments calculated using a certain convention while others have used different conventions. Although

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these mundane details have their roots in history, they effect our everyday life by determining the
amount we pay on our liabilities and the amount we are owed on our assets.

In many situations, there is no clear reason why one day-count convention would be preferred
over another. Conventions that use a fixed number of days per month allow for fixed monthly
interest payments; conventions based solely on actual number of elapsed calendar days may be
most intuitive but lead to fluctuations in the interest due each month; conventions that fix the
number of days per year eliminate leap-year effects but can still cause interest payments to vary
by month. The wide variety of day-count conventions in practice today developed from the need
for lenders and issuers to coordinate day-count conventions of specific obligations.

1.1 Day-Count Convention Definitions

We focus on the six of the most common day-count conventions. Day-count conventions are
typically written as a fraction with the numerator representing the number of days elapsed per
month and the denominator representing the number of days elapsed per year. Assume we are
trying to determine what year-fraction we should use to determine the interest accrued between a
calendar date given by \( M_1/D_1/Y_1 \) and another calendar date given by \( M_2/D_2/Y_2 \).

The six day-count conventions we consider can be grouped into two categories based upon
the way the numerator is calculated. These first three day-count conventions assume each month
consists of 30 days and each year consists of twelve 30-day months for the calculation of the
numerator. More explicitly, the number of days between two calendar days is given by

\[
N = (12)(30)(Y_2 - Y_1) + 30(M_2 - M_1) + (D_2 - D_1). \tag{1}
\]

The following three day-count conventions use this procedure to calculate the numerator in an
interest crediting formula.

- **30/360**: This convention assumes that each month is of equal length (30 days) and that the
  year is 360 days long. The fraction of a year over which interest is accrued between these
two calendar dates is \( N/360 \)[1]. For example, if \( M_1/D_1/Y_1 = 06/17/2011 \) and \( M_2/D_2/Y_2 =
12/30/2012 \), then \( N = (12)(30)(2012 - 2011) + 30(12 - 6) + (30 - 17) = 553 \). As a result,
the year fraction used to calculate interest payments is \( 553/360 = 1.5361 \).

- **30/365**: This convention assumes that each month is of equal length (30 days) and that the
  year is 365 days long. The number of days between two calendar days is given by Equation
[1] The fraction of a year over which interest is accrued between these two calendar dates is \( N/365 \). For example, if \( M_1/D_1/Y_1 = 06/17/2011 \) and \( M_2/D_2/Y_2 = 12/30/2012 \), then
\( N = (12)(30)(2012 - 2011) + 30(12 - 6) + (30 - 17) = 553 \). As a result, the year fraction
used to calculate interest payments is \( 553/365 = 1.5151 \).

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[1] There is a variety of day-count conventions that fall under this category and the difference amongst them can mainly be attributed to their treatment of the case in which \( D_1 \) or \( D_2 \) are greater than or equal to 29.

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• **30/ACT:** Similar to the previous two conventions, this convention assumes each month is 30 days long and that the number of days elapsed is computed using Equation 1. \( N \) is then segmented into the number of days within a leap year \( (N_L = 360) \) and the number of days in a non-leap year \( (N_{NL} = 193) \). The year fraction used to compute interest accrued is then given by \( \frac{360}{366} + \frac{193}{365} = 1.5124. \)

The second category involves day-count conventions in which the numerator is calculated as the actual number of days elapsed between two calendar dates. The following three day-count conventions compute the actual number of days elapsed to compute the numerator in an interest crediting formula.

• **ACT/360:** This day-count convention determines the actual number of days between two calendar dates and divides the result by 360 to determine the year fraction. For example, if \( M_1/D_1/Y_1 = 06/17/2011 \) and \( M_2/D_2/Y_2 = 12/30/2012 \), then \( N = (30 - 17) + 31 + 31 + 30 + 31 + 30 + 31 + 365 = 562 \) and the year fraction is \( \frac{562}{360} = 1.5611. \)

• **ACT/365:** This day-count convention determines the actual number of days between two calendar dates and divides the result by 365 to determine the year fraction. For example, if \( M_1/D_1/Y_1 = 06/17/2011 \) and \( M_2/D_2/Y_2 = 12/30/2012 \), then \( N = (30 - 17) + 31 + 31 + 30 + 31 + 30 + 31 + 365 = 562 \) and the year fraction is \( \frac{562}{365} = 1.5397. \)

• **ACT/ACT:** This day-count convention determines the actual number of days between two calendar dates \( (N) \) and then segments into two parts: the number of days within a leap year \( (N_L) \) and the number of days in a non-leap year \( (N_{NL}) \). For example \( M_1/D_1/Y_1 = 06/17/2011 \) and \( M_2/D_2/Y_2 = 12/30/2012 \), then \( N_L = 365 \) and \( N_{NL} = (30 - 17) + 31 + 31 + 30 + 31 + 30 + 31 + 365 = 197 \) and therefore the year fraction is given by \( \frac{365}{366} + \frac{197}{365} = 1.5370. \)

Table 1 gives several examples of the fraction of a year used to calculate interest accrued over different calendar months. For the table, we assume each calendar year indicates a separate year for interest calculations to simplify exposition.

The fraction of a year used to compute coupon payments varies from a high of 0.08611 within the ACT/360 convention to a low of 0.07671 for the ACT/365 and ACT/ACT conventions. A coupon payment based on a fixed rate of 5% on a notional amount of $10,000,000 could vary between \( 10,000,000 \times 5\% \times 31/360 = 43,055.56 \) and \( 10,000,000 \times 5\% \times 28/365 = 38,356.16 \) depending on the day-count convention used to calculate the interest payment. In fact, if one uses the ACT/ACT convention the fixed rate payment could vary by more than 9%.

Table 1 indicates a strict preference order for the interest payer: the higher the day-count fraction the larger his or her interest rate payments. To be explicit, an investor with a fixed-rate obligation would have the following order for their preferences: 30/ACT, 30/365, 30/360, 30/360, 30/360, 30/360, 30/360.

\[21 - 0.07671/0.08470 = 9.4\%]
Table 1: Fraction of a year used to compute coupon payments for various day-count conventions

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ACT/360</td>
<td>0.08611</td>
<td>0.08056</td>
<td>0.07778</td>
<td>0.08333</td>
</tr>
<tr>
<td>ACT/365</td>
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<td>0.07945</td>
<td>0.07671</td>
<td>0.08219</td>
</tr>
<tr>
<td>30/360</td>
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<td>0.08333</td>
<td>0.08333</td>
<td>0.08333</td>
</tr>
<tr>
<td>ACT/ACT</td>
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<td>0.07923</td>
<td>0.07671</td>
<td>0.08197</td>
</tr>
<tr>
<td>30/365</td>
<td>0.08219</td>
<td>0.08219</td>
<td>0.08219</td>
<td>0.08219</td>
</tr>
<tr>
<td>30/ACT</td>
<td>0.08197</td>
<td>0.08197</td>
<td>0.08219</td>
<td>0.08197</td>
</tr>
</tbody>
</table>

ACT/ACT, ACT/365, ACT/360 \(^3\) An individual with a fixed-rate asset would have the opposite preference order.

In Table 2, we present the ratio of a given day-count factor to another day-count factor for a full calendar year. Table 2 summarizes the results for non-leap year and leap years separately. These results demonstrate that the change from one day-count convention can change the interest accrual factor by as much as 3%.

Table 2: Ratio of day-count factors between day-count conventions

<table>
<thead>
<tr>
<th></th>
<th>ACT/360</th>
<th>ACT/365</th>
<th>30/360</th>
<th>ACT/ACT</th>
<th>30/365</th>
<th>30/ACT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-leap Years</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ACT/360</td>
<td>100%</td>
<td>101.4%</td>
<td>101.4%</td>
<td>101.4%</td>
<td>102.8%</td>
<td>102.8%</td>
</tr>
<tr>
<td>ACT/365</td>
<td>98.6%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>101.4%</td>
<td>101.4%</td>
</tr>
<tr>
<td>30/360</td>
<td>98.6%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>101.4%</td>
<td>101.4%</td>
</tr>
<tr>
<td>ACT/ACT</td>
<td>98.6%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>101.4%</td>
<td>101.4%</td>
</tr>
<tr>
<td>30/365</td>
<td>97.3%</td>
<td>98.6%</td>
<td>98.6%</td>
<td>98.6%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>30/ACT</td>
<td>97.3%</td>
<td>98.6%</td>
<td>98.6%</td>
<td>98.6%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Leap Year</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ACT/360</td>
<td>100%</td>
<td>101.4%</td>
<td>101.7%</td>
<td>101.7%</td>
<td>103.1%</td>
<td>103.4%</td>
</tr>
<tr>
<td>ACT/365</td>
<td>98.6%</td>
<td>100%</td>
<td>100.3%</td>
<td>100.3%</td>
<td>101.7%</td>
<td>101.9%</td>
</tr>
<tr>
<td>30/360</td>
<td>98.4%</td>
<td>99.7%</td>
<td>100%</td>
<td>100%</td>
<td>101.4%</td>
<td>101.7%</td>
</tr>
<tr>
<td>ACT/ACT</td>
<td>98.4%</td>
<td>99.7%</td>
<td>100%</td>
<td>100%</td>
<td>101.4%</td>
<td>101.7%</td>
</tr>
<tr>
<td>30/365</td>
<td>97.0%</td>
<td>98.4%</td>
<td>98.6%</td>
<td>98.6%</td>
<td>100%</td>
<td>100.3%</td>
</tr>
<tr>
<td>30/ACT</td>
<td>96.7%</td>
<td>98.1%</td>
<td>98.4%</td>
<td>98.4%</td>
<td>99.7%</td>
<td>100%</td>
</tr>
</tbody>
</table>

\(^3\)30/360 and ACT/ACT are essentially the same for every year that is not a leap year for a fixed-rate obligation with no discounting of cash-flows.
1.2 Common Uses of Day Count Conventions

Different types of securities use different day-count conventions, which can affect the comparison of their yields. For example, US corporate and municipal bonds typically use the 30/360 convention, whereas US Treasury bonds typically use ACT/ACT and money market agreements often use ACT/360. European (including British) securities typically adhere to the ACT/ACT convention. Australian money-market securities use ACT/365. However, beyond traditional bond securities, day-count convention usage can vary widely even for similar securities, and the already limited standards that do exist can and have changed even in the last decade.[4]

Guidance on day-count conventions comes from many regulatory and industry sources. The Financial Industry Regulatory Authority (FINRA)'s Uniform Practice Code specifies the 30/360 convention to be used “in the settlement of contracts in interest-paying securities other than for ‘cash’.”[5] The International Capital Market Association requires ACT/360 for floating rate notes and USD-denominated straight and convertible bonds, but ACT/ACT for non-USD straight and convertible bonds.[6] The Municipal Securities Rulemaking Board (MSRB) specifies 30/360 in Rule G-33(e) regarding “municipal securities transactions and for municipal advisors that engage in municipal advisory activities.”[7] SIFMA, the Securities Industry and Financial Management Association, has a publication describing a variety of day-count conventions, including the official definition of the 30U/360 and ACT/ACT (ICMA) conventions.[8] Finally, the International Swaps and Derivatives Association (ISDA) defines all day-count conventions used for swap agreements in their 2006 ISDA Definitions publication.

Many textbooks on fixed-income securities have a section on calculating day-count factors and converting between conventions. Fabozzi’s “Handbook of Fixed Income Securities” only explains ACT/ACT and 30/360 conventions.[9] Hull’s derivatives textbook describes both the most common conventions, and explains why many fixed-leg calculations rely on the 30/360 convention (to keep fixed payments fixed month-to-month).[10] He also includes an insert, titled “Day Counts Can be Deceptive”, in which he describes the difference in interest payments that would result from a US Treasury bond versus a US corporate bond when held at the end of February on a non-leap year. These discussions are helpful, but limit their scope to the bond market, and may not be sufficient guidance for purchasers of swaps.

Over-the-counter agreements (including swap contracts) have no single convention even for particular types of agreements. Because these contracts’ choice of day-count convention vary, it

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[4] See citations below from regulatory agencies, as well as [Hull, 2011] and the ISDA Memo on European Swaps (ISDA - BS:9951.1). Interestingly, some quantitative analysts build models according to an ACT/365.25 convention; see [Andersen and Piterbarg, 2010].


[6] ICMA Rule 251.1


is critical for investors to understand the different conventions and their impact on interest rate payments. While a dealer may suggest that a particular convention is common or standard practice, a disadventageous choice of convention could lead to what amounts to an undisclosed transfer fee on what are typically large, long-lived agreements. In the following section we demonstrate how to value day-count conventions within interest-rate swap agreements.

2 Analysis of Day-Count Conventions

2.1 Plain Vanilla Interest Rate Swaps

The market value of an interest rate swap to the fixed rate receiver is the present value of the fixed-rate payments less the present value of the variable rate payments at the time the deal is struck. More explicitly,

\[ V_{\text{Swap}} = PV_{\text{fixed}} - PV_{\text{variable}}. \]

when the agreement is priced. Consider a swap with notional value \( P \) and let \( c \) be the fixed interest rate on the swap. Let \( \Delta t^f_i \) be the number of days in period \( i \) and let \( T^f_i \) be the basis on which interest is calculated for the fixed leg. The present value of the fixed rate leg is then given by

\[ PV_{\text{fixed}} = \sum_{i=1}^{N} (cP) \left( \frac{\Delta t^f_i}{T^f_i} \right) df_i, \]  

where \( df_i \) is the discount factor for payment \( i \) and \( N \) payments are made throughout the swap. Let \( r_i \) be the variable rate for period \( i \) and let \( T^v_i \) be the basis on which interest is calculated for the floating leg. The present value of the variable rate leg is given by

\[ PV_{\text{variable}} = \sum_{i=1}^{N} (r_iP) \left( \frac{\Delta t^v_i}{T^v_i} \right) df_i. \]

Of course the value of the variable rate leg is highly dependent on expectations of future interest rates. The discussion within this paper is independent of the model chosen since we focus on variations in day-count conventions of interest rate swaps.

The day-count convention in the swap agreement determines both \( \Delta t^v_i; f \) and the basis \( T^v_i; f \) and these are in general different for the two legs. The day-count conventions can have a dramatic effect on the value of a swap since they determine the proportion of the interest rate in each period that should be applied. To illustrate this point, consider the ratio of the payments to be exchanged at payment date \( i \)

\[ R_i = \frac{(cP) \left( \frac{\Delta t^f_i}{T^f_i} \right) df_i}{(r_iP) \left( \frac{\Delta t^v_i}{T^v_i} \right) df_i} = \left( \frac{c}{r_i} \right) \left( \frac{T^v_i}{T^f_i} \right) \left( \frac{\Delta t^f_i}{\Delta t^v_i} \right). \]
**Example:** If the fixed leg is calculated on a 30/360 basis and the floating leg is calculated on an ACT/360 basis, for example, then the variable rate is effectively \( r_i \times \frac{\Delta t^v_i}{\Delta t^f_i} \) which is greater than or equal to \( r_i \) for all months other than February. As a result, the variable interest rate is effectively higher (or the fixed rate is effectively lower) as a result of the disparity in day-count conventions.

**Example:** If the fixed leg is calculated on an ACT/365 basis and the floating leg is calculated on a ACT/360 basis, for example, then the fixed rate is effectively \( c \times 360/365 = 0.986c \). As a result, the fixed interest rate is effectively lower as a result of the disparity in day-count conventions.

### 2.2 Matching Obligations

Interest rate swaps are often used to alter the level of interest rate risk two parties experience as a result of the assets or liabilities currently on their books. For example, if a borrower owes a lender LIBOR + 1.7% then entering into an interest rate swap where the borrower receives LIBOR + 1.7% and pays a fixed rate effectively creates a fixed rate obligation. See Figure 1 for a graphical depiction of the payments exchanged between the parties involved each pay period.

**Figure 1:** Example of a plain vanilla interest rate swap used to change a floating rate obligation to a synthetic fixed rate obligation

Consider a situation wherein a small-business owner takes out a $10 million fixed rate loan with rate 5% and ACT/360 day-count convention such that the principal amount will be fully amortized in 5 years. Suppose that the small-business owner also enters into an amortizing interest rate swap as the fixed-rate receiver – with a notional amount determined by the amortization schedule of the loan. The terms of the swap are such that the business owner pays a variable rate and receives 5% based upon the 30/360 day-count convention.
If the loan and swap begin on January 1, 2012, the interest on the first month of the loan is $10,000,000 \times 5\% \times 31/360 = 43,055.6$; however, according to the terms of the swap agreement, the small-business owner is to receive $10,000,000 \times 5\% \times 30/360 = 41,666.67$. So this difference in day-count conventions amounts to an additional spread on the variable rate leg of the swap for which the small-business owner is responsible. Discounting the cash flows back by a constant 3% simple interest (30/360), the difference in day-count conventions is equivalent to an upfront fee of about $17,960 – an 18 basis point fee on the initial loan principal.

This difference could be large enough to turn the value of a swap from positive to negative. Indeed, 18 basis points could be larger than the yearly fee assessed by the financial intermediary in a plain vanilla swap. Total fees on 10 year swaps are typically 0.5%–1% of notional.\(^{11}\) Therefore, day-count convention alone could increase total fees by over 20%.

Our conclusion, therefore, is that a fairly priced interest rate swap using one convention is almost certainly mispriced using another convention. In the best case, this often overlooked parameter can be used by each party to appropriately match the cash flows of swapped obligations. In the worst case, this seemingly innocuous parameter can be used by the financial intermediary to collect yet another fee.

### 3 Valuing Day-Count Effects Using Market Data

In this section, we highlight the influence of day-count conventions on the market value of interest rate swaps. We used Bloomberg to value plain-vanilla interest rate swaps in which monthly floating rate payments based upon 1-month LIBOR (computed on an ACT/360 basis) were exchanged for monthly fixed rate payments. In order to simplify the discussion, we have assumed that there is no fixed spread added to the floating leg of the agreements, that each agreement is based upon the constant $10,000,000 notional amount and that each agreement is entered into and traded on January 1 of the first year of the swap.

To ease the comparison between the swap agreements with identical terms except the day-count convention used to accrue interest on the fixed leg of the agreement, we determine the par coupon for the swap agreement with a 30/360 day-count convention applied to the fixed leg of the agreement.\(^{12}\) We then change only the day-count convention used to accrue interest on the fixed leg and determine the market value of such an agreement as well as the par coupon within this new day-count convention.

The market value quoted for an agreement with a day-count convention different from the 30/360 convention represents the fee one party pays another in the event a different day-count convention is applied but the rate is not changed to compensate for this change. If the market

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\(^{11}\) Whittaker, March 1987

\(^{12}\) The \textit{par coupon} is the fixed interest rate on the fixed leg of the agreement that results in a zero market value for the swap agreement. In other words, it is the coupon payment that results in the present value of fixed leg payments equaling the present value of the floating rate payments.
value is positive, then the swap is more favorable to the fixed-rate payer and the market value effectively represents an upfront payment from the floating-rate payer to the fixed-rate payer. If the market value is negative, then the swap is more favorable to the floating-rate payer and the market value effectively represents an upfront payment from the fixed-rate payer to the floating-rate payer.

3.1 Dependence on Swap Term

In Figure 2, we plot the market value as a percentage of notional value versus the term of the underlying swap agreement. All swap agreements were valued on January 1, 2012 and we considered terms varying from 5 years to 20 years.

Figure 2: Difference in market value of swap agreement as a percentage of notional value when the day-count convention of the fixed leg is changed (keeping the fixed rate unchanged) as a function of the term of the swap agreement

The market value of the agreement is roughly a linear function of the term. For example, the fixed-rate payer in a five year swap traded on January 1, 2012 that uses an 30/ACT rather than 30/360 for the fixed-leg of the swap agreement has essentially paid an upfront fee of approximately 8bps (1bp = 0.01%) to the floating-rate payer. If the term had been ten years, the fixed-rate payer would have essentially paid an upfront fee of 27bps.

3.2 Dependence on Swap Starting Date

In Figure 3, we plot the value of a one year swap entered into at the beginning of each year between 2000 and 2012. This effectively changes the interest rate term structure and market values as of the trade date.
Figure 3: Difference in market value of swap agreement as a percentage of notional value when the day-count convention of the fixed leg is changed (keeping the fixed rate unchanged) as a function of the starting date of the swap agreement.

The market value of different day-count conventions used in the same one year vanilla swap agreement can vary considerably over time: a $10 million notional ACT/360-based swap initiated on January 1, 2000 would be worth approximately $10,241.66 more to the investor than an otherwise identical 30/ACT swap; the difference on January 1, 2009 would be only $1,046.14. This difference in value caused by misaligned day-count conventions is not simply a function of the initial interest rates, but expectations of future interest rates further out on the term structure (in this case, up to 1 year). In Figure 3 we also plot a simple metric for the shape of the term structure, which is 1-year LIBOR divided by 1-month LIBOR minus one. When the term structure is relatively steep (a large expected increase in interest rates), the difference in day-count conventions has a smaller impact on the market value than when the term structure is less steep.

3.3 Dependence on Payment Frequency

Table 3 shows the relative value of each convention for weekly, monthly, quarterly, semiannual, and annual payment frequencies. These results show the relative gain or loss of value relative to a 30/360 fixed-leg swap as a fraction of the total notional value (in basis points), for each payment frequency. The effect of a misapplied day-count convention is generally amplified for swaps with lower-payment frequencies.

3.4 Dependence on Market Value

We find that the effect of the changing day-count conventions is insensitive to the magnitude of the initial fee charged by the counterparty for structuring the agreements. Table 4 summarizes...
Table 3: Difference in market value of swap agreement in basis points of notional value when the day-count convention of the fixed leg is changed (keeping the fixed rate unchanged) as a function of the payment frequency of the swap agreement

<table>
<thead>
<tr>
<th>Convention</th>
<th>Weekly</th>
<th>Monthly</th>
<th>Quarterly</th>
<th>Semiannual</th>
<th>Annual</th>
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</thead>
<tbody>
<tr>
<td>ACT/360</td>
<td>23.6</td>
<td>26.7</td>
<td>27.6</td>
<td>29.2</td>
<td>32.2</td>
</tr>
<tr>
<td>ACT/365</td>
<td>-1.6</td>
<td>1.4</td>
<td>1.5</td>
<td>1.6</td>
<td>1.8</td>
</tr>
<tr>
<td>ACT/ACT</td>
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<td>-0.1</td>
<td>-0.1</td>
<td>-0.1</td>
<td>0</td>
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<tr>
<td>30/365</td>
<td>-24.9</td>
<td>-24.9</td>
<td>-25.7</td>
<td>-27.2</td>
<td>-29.9</td>
</tr>
<tr>
<td>30/ACT</td>
<td>-26.4</td>
<td>-26.4</td>
<td>-27.3</td>
<td>-28.9</td>
<td>-31.7</td>
</tr>
</tbody>
</table>

this result by showing the percent change in fee charged to the fixed payer when the day-count convention is altered but the fixed rate is not adjusted.

Table 4: Percent change in market value (to fixed-rate receiver) when the day-count convention is changed (the fixed rate is not changed)

<table>
<thead>
<tr>
<th>Market Value</th>
<th>ACT/360</th>
<th>ACT/365</th>
<th>ACT/ACT</th>
<th>30/365</th>
<th>30/ACT</th>
</tr>
</thead>
<tbody>
<tr>
<td>3%</td>
<td>-7.44%</td>
<td>-1.40%</td>
<td>0.02%</td>
<td>6.94%</td>
<td>7.35%</td>
</tr>
<tr>
<td>2%</td>
<td>-11.89%</td>
<td>-0.64%</td>
<td>0.03%</td>
<td>11.10%</td>
<td>11.75%</td>
</tr>
<tr>
<td>1%</td>
<td>-25.26%</td>
<td>-1.35%</td>
<td>0.07%</td>
<td>23.56%</td>
<td>24.96%</td>
</tr>
<tr>
<td>-1%</td>
<td>-28.19%</td>
<td>-1.51%</td>
<td>0.08%</td>
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<td>27.86%</td>
</tr>
<tr>
<td>-2%</td>
<td>-14.83%</td>
<td>-0.79%</td>
<td>0.04%</td>
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<td>14.65%</td>
</tr>
<tr>
<td>-3%</td>
<td>-10.71%</td>
<td>-0.56%</td>
<td>0.03%</td>
<td>9.68%</td>
<td>10.25%</td>
</tr>
</tbody>
</table>

The percentage change in market value of a swap due to a misapplied day-count convention is inversely proportional to the initial swap fee charged. For example, a change from 30/360 to 30/365 results in an approximate 24% increase in market value to the fixed rate payer when the fee is 1% of notional but only 11% when the fee is 2% of notional – approximately half as much. As a result, the effect of a misapplied day-count convention is more significant the lower the fee and depends most significantly on the notional of the swap agreement.

3.5 Summary

We find that day-count effects are nearly linearly dependent on the term of the agreement and strongly dependent on the term structure of expected future interest rates. We also observe an effect of payment frequency, whereby higher payment frequencies generally decrease day-count discrepancies. We find that day-count effects are relatively unaffected by the baseline market value of the agreement and are proportional to notional value.

In general, factors which increase the amount or frequency of interest payments (frequency, term) amplify day-count effects, as could be expected from our results in Section 2. The notional
amount or fees may affect the market value of an agreement but do not affect the proportion of market value differences attributable to changes in day-count conventions. We also find that the level of current interest rates is not as important as the term structure of interest rates, which reflect the entire range of future interest payments over the term of the deal.

4 Discussion

This paper studies one of the most overlooked parameters of interest-bearing securities: day-count conventions. We find that changing the day-count convention in a swap agreement without appropriately compensating the coupon payment can result in large changes in the market value of the agreement, and can even lead to an asset for one party changing to a liability.

We showed the importance of matching cash-flows of obligations/assets with associated swap agreements and how the mismatch of day-count conventions can lead to disadvantageous wealth transfers. We demonstrated the sensitivity of day-count convention effects to the pricing date and term of a swap agreement, noting in particular that the term structure of interest rates (as opposed to current levels of rates) can have a large effect on the magnitude of day-count effects.

We focus on interest rate swap agreements in particular because these agreements are often used by institutions such as school districts, municipalities, and other public authorities who have large capital bases and needs but may lack sufficient expertise to negotiate such agreements carefully. These day-count effects are not limited to swap agreements, but are a property of any interest-accruing security. At the very least, investors should be aware of the preference order for day-count conventions implied from Table 1, as well as the parameters in Section 2 which could influence the magnitude of day-count effects. Our results suggest that purchasers of interest rate swaps should include day-count effects into their valuation models and assess their impact before entering into agreements, as a simple change of convention could lead to a large effective market value loss.

References


