The Anatomy of Principal Protected Absolute Return Barrier Notes

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Abstract

Principal Protected Absolute Return Barrier Notes (ARBNs) are structured products linked to an underlying security or an index. While these notes guarantee principal protection – return of face value – their upside potential is dependent on the level of the underlying security never falling outside of a predefined range. This, combined with the credit risk of the issuer to which all structured products are subject, makes these products difficult to value.

In this paper we value ARBNs by decomposing the note into a zero coupon bond and double barrier linear segment options. We derive closed form solutions for ARBNs and their Greeks, then value 214 publicly-listed ARBNs issued by six different investment banks between 2006 and 2009. We find that the ARBNs' fair price is approximately 4.5% below the actual issue price on average.

1 Introduction

Structured products are complex debt instruments whose payoffs are linked to the performance of reference stocks, indices, commodity prices, interest rates, or exchange rates. Structured products have grown in popularity in the past decade. At the same time, the

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new products offered tend to exhibit an increasing level of complexity (Hernández et al., 2007; Henderson and Pearson, 2010), especially, by offering investors explicit or implicit exposure to all, some, or none of the underlying security's downside risk.

A Principal Protected Absolute Return Barrier Note (ARBN) is a structured product that is designed to limit investor's downside losses through its 'principal protection' feature. However, like all other structured products, an ARBN exposes investors to the default risk of the issuer, and more importantly, ARBNs expose investors to the upside potential of the reference security only if certain criteria are met. Therefore, like many other structured products, the underlying value of ARBNs is not intuitive or easily calculated.

Fortunately, many papers have started exploring the characteristics and valuations of specific types of structured products. For example, Hernández et al. (2007) and Szymanowska et al. (2009) all explore reverse convertibles. Reverse convertibles, one of the most analyzed structured product classes, are products that can be converted from a debt instrument into the underlying security at the option of the issuer. All the research on reverse convertibles shows a substantial premium on the issue date. For example, Henderson and Pearson (2010) analyze SPARQS, a type of reverse convertible that is callable by the issuer on scheduled call dates. Like all reverse convertibles, SPARQS tend to be issued at a premium over fair value: Henderson and Pearson (2010) find that *"reasonable estimates of the expected returns on SPARQS are less than the riskless rate. For the estimates of expected returns on the underlying stocks that seem most reasonable, the average expected return on the SPARQS is actually negative"* (p. 30).

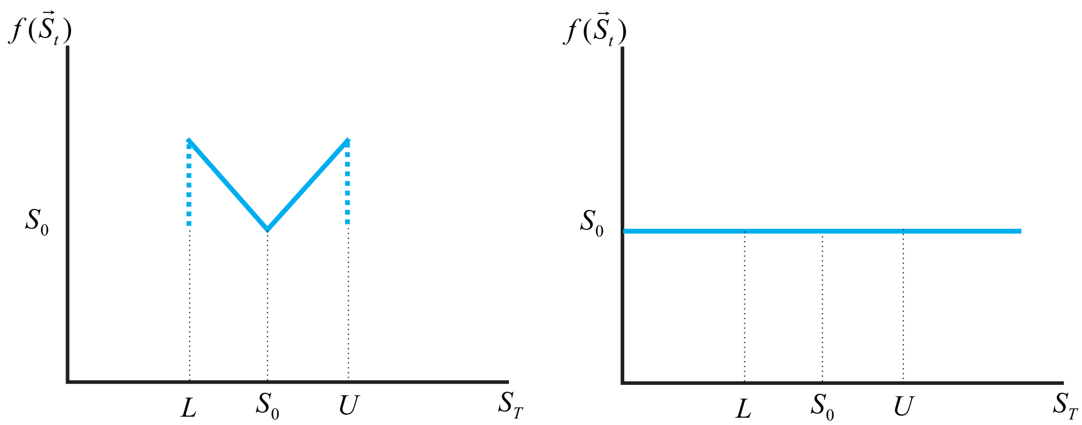
Like reverse convertibles, ARBNs have grown substantially in the past few years – 2008 saw the issuance of more than \$1.6b worth. ARBNs have been sold to the public highlighting two main features. First, their perceived principal protection. As Keith Styrcula, the chairman of the Structured Products Association argued that *"[ARBNs are] an opportunity to get an above market return with protection. You either get everything or nothing but your principal."*[1] Second, their perceived limited exposure to volatility risk. For example, a BusinessWeek article about ARBNs argued that *"With so much uncertainty swirling, some money managers are pushing instruments designed to limit investors' exposure to volatility. .... Given the way the market has been performing, just treading water may be enough for many investors."* (Goldstein et al. (2008))

The principal protection feature guarantees the full payback of the note's face value as long as the investor holds the note to maturity and the issuer does not default on the note. The interest portion of the ARBN's payoff at maturity, however, is conditional on the entire return path of the underlying security. If the price of the underlying security remains within a lower and an upper barrier (L and U, respectively) for the entire life of the note, the interest included in the payoff at maturity is equal to the absolute value of

[1] "Stocks: More Doldrums Ahead", BusinessWeek, July 2, 2008.

the underlying security's return. If the price of the underlying security ever crosses the lower or upper barrier, the note does not pay interest. Figure 2(a) and Figure 2(a) graphs the general payoff structure of an ARBN in two cases: in Panel A we describe the payoff conditional its underlying security not having breached a barrier. In Panel B we describe the payoff in the case that the underlying security breached a barrier at least once.

Figure 1: Payoff Structure ($f(\vec{S}_t)$) of an ARBN conditional on not having breached a barrier. L and U are the lower and upper barriers, respectively. S_0 is the initial level of the underlying security. Since the payoff structure is path dependent, we introduce the notation $\vec{S}_t$ to represent the price trajectory of S_t over time $[0, T]$. When the stock price $\vec{S}_t$ remains within the barriers for its entire trajectory, the note receives positive interest. The principal protection feature is indicated by the horizontal line below L and above U.



(a) Stock price stays in barriers

(b) Otherwise

The pattern of ARBNs' conditional interest payment has similarities to double-barrier options (Carr et al., 1998; Li, 1998; Davydov and Linetsky, 2001). Double-barrier options, which are one of the most popular over-the-counter options (Carr and Crosby, 2008), include both a lower barrier L and an upper barrier U. The function of the barriers depends on whether the option is *knock-in* or *knock-out*. Knock-in double-barrier options cannot be exercised unless the underlying security's price crosses either of the two barriers during the option's contract. In contrast, similarly to ARBNs, knock-out double barrier options *lose* their exercise ability if the underlying security's price crosses either of the two barriers. Indeed, applying the techniques used to value double barrier options gives us a convenient way of evaluating the conditional aspect of ARBN payoffs.

We use this approach to derive a closed-form valuation of ARBNs. We apply the valuation to 214 ARBNs issued by six investment banks: Deutsche Bank, Goldman Sachs, HSBC, Lehman Brothers, Morgan Stanley, and UBS. We model the price of each ARBN at issuance, and find that the products have a 4.5% issue premium on average relative to the price we model. The implied yield for these products is generally lower than the issuer's corporate yield, and in some cases, it is even lower than the risk-free rate. We analyze and summarize the actual returns of all the matured ARBNs.

The paper is organized as follows. In Section 2 we derive the closed-form valuation equations as well as hedging analysis. In Section 3 we value ARBNs from 6 investment banks in the market. We conclude in Section 4.

2 Valuation and Hedging of the Notes

In this section, we describe how to value and hedge ARBNs. Section 2.1 introduces overall assumptions in our valuations. Section 2.2 presents the valuation by decomposition via double-barrier linear segment options (DBLS). We use DBLS options as the component options for their simplified forms in representing the solution, i.e., the equational forms of call and put options are unified. A method for delta hedging of ARBNs is also presented in Section 2.3. We also analyze graphically the valuation of an ARBN and the behavior of its Greeks: delta and gamma.

2.1 Assumptions

We use similar assumptions as in the Black-Scholes model for option valuation. We assume the underlying security price S_t follows a geometric Brownian Motion

$$dS_t = \mu S dt + \sigma S dZ_t, \tag{1}$$

where, under the risk-neutral measure, μ is a constant drift defined as

$$\mu = r - q.$$

r is the risk free rate and q is the dividend yield of the underlying security.[2] σ is the volatility of the process and Z_t is a standard Brownian motion. At maturity ($t = T$), the underlying security's price, conditioned on the underlying security's initial price S_0, is log-normally distributed

$$g(S_T|S_0) = \text{Log-}\mathcal{N}\left(\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right), \tag{2}$$

[2]We assume both the risk free rate and the dividend yield are constant over time in the model.

where $g(\cdot)$ denotes a probability density function.

At maturity, ARBN returns the face value to the investors. In addition, if the underlying security's price remains within the barriers $[L, U]$ for all $t \in [0, T]$, the ARBN pays a return equal to the absolute value of the underlying security's return. For simplicity, we introduce two timing variables denoting the first time the stock breaches a barrier: τ_L and τ_U . They are defined as

$$\tau_L = \inf\{t |\, S_t = L\}$$

and

$$\tau_U = \inf\{t |\, S_t = U\}.$$

Thus, the ARBN pays a return above the face value of the note only if $\min(\tau_L, \tau_U) > T$. The ARBN payoff function, $f(\vec{S}_t)$, is written as

$$f(\vec{S}_t) \quad = \quad \begin{cases} S_0 + |\frac{S_T - S_0}{S_0}|, & \text{when } \min(\tau_L, \tau_U) > T, \\ S_0, & \text{otherwise.} \end{cases} \tag{3}$$

f is a function of $\vec{S}_t$, indicating that the payoff of the note relies on the historical prices of the underlying security.

2.2 Valuation by Decomposition

In a general decomposition approach, the structured note's payoff is broken down into an equivalent portfolio of simple bond instruments, option contracts, forward contracts, and swaps, and have a closed-form valuations.

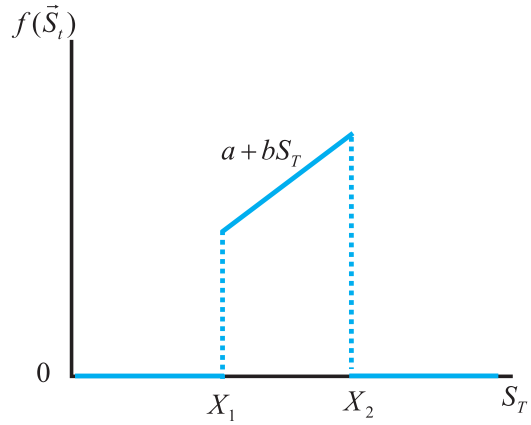
We decompose the price of an ARBN into a zero-coupon bond and two knock-out, double-barrier linear segment options (DBLS).[3] Other decompositions are valid as well, such as double barrier call and put options. We use DBLS as they allow for simple and clean representation of the result. A knock-out DBLS pays $a + bS_T$ when the final equity price S_T falls inside the interval $[X_1, X_2]$ and the prices $\vec{S}_t$ remain in the barriers, and 0 otherwise. The final payoff of a knock-out DBLS when neither barrier is breached is shown graphically in Figure 2. Considering the same barriers L and U, the functional form is represented as:

$$f_{DBLS}(\vec{S}_t, a, b, X_1, X_2) \quad = \quad \begin{cases} a + bS_T, & \text{when } \min(\tau_L, \tau_U) > T \text{ and } S_T \in [X_1, X_2], \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

A DBLS without barriers, called a linear segment option (LS), is a generalized option from which other options are derived as special cases. For example, a binary option is

[3] See Li (1998) for a more detailed discussion of double-barrier linear segment options.

Figure 2: Payoff mapping function of DBLS with parameters (a, b, X_1, X_2), when the barriers L and U are not breached.



a LS option with parameters $(a, b, X_1, X_2) = (1, 0, X_1, X_2)$; a call option is a LS option with parameters $(a, b, X_1, X_2) = (-S_0, 1, S_0, \infty)$; and a put option is a LS option with parameters $(a, b, X_1, X_2) = (S_0, -1, 0, S_0)$. The payoff function for LS option is simply

$$f_{LS}(S_T, a, b, X_1, X_2) = a + bS_T, \text{ when } S_T \in [X_1, X_2],$$

which relies only on final stock price S_T.

The payoff of an ARBN can be decomposed as

$$f_{ARBN}(\vec{S}_t) = S_0 + f_{DBLS}(\vec{S}_t, -S_0, 1, S_0, U) + f_{DBLS}(\vec{S}_t, S_0, -1, L, S_0). \quad (5)$$

The equation indicates that the ARBN payoff is equivalent to a portfolio containing the three following securities: a) 1 share of a zero-coupon bond with a face value of S_0, b) 1 share of a knock-out DBLS with parameters $(a, b, X_1, X_2) = (-S_0, 1, S_0, U)$, and c) 1 share of a knock-out DBLS with parameters $(a, b, X_1, X_2) = (S_0, -1, L, S_0)$. Thus, the ARBN's fair value on issue date is derived as:

$$V_{ARBN}(\vec{S}_t) = e^{-(r+\bar{C})T} S_0 + V_{DBLS}(\vec{S}_t, -S_0, 1, S_0, U) + V_{DBLS}(\vec{S}_t, S_0, -1, L, S_0). \quad (6)$$

We include the credit default swap (CDS) spread $\bar{C}$ of the issuer in the discount factor. Including the CDS spread adjusts the valuation for the counterparty risk faced by the investor.[4]

[4]See (Hull, 2008; Jarrow and Turnbull, 1995) on how to adjust derivative prices for counterparty credit risk. Credit risk affects the prices of structured products. This is reflected anecdotally by the fact that prices of Bear Stearns structured products dropped significantly in March 2008, and bounced back when JP Morgan announced its acquisition.

Li (1998) provides the following valuation formula for the knock-out DBLS

$$\begin{aligned} V_{DBLS}(\vec{S}_t, a, b, X_1, X_2) &= \sum_{n=-\infty}^{\infty} \left[V_{LS}\left(S_0 \left(\frac{U}{L}\right)^{2n}, a, b, X_1, X_2 \right) - \right. \\ & \left. V_{LS}\left(\frac{U^2}{S_0} \left(\frac{U}{L}\right)^{2n}, a, b, X_1, X_2 \right) \left(\frac{U}{S_0}\right)^{\frac{2\lambda}{\sigma^2}} \right] \left(\frac{U}{L}\right)^{\frac{2n\lambda}{\sigma^2}}. \end{aligned}$$

λ is a constant equal to $r - q - \sigma^2/2$. The standard linear segment option value V_{LS} is given as

$$\begin{aligned} V_{LS}(S_0, a, b, X_1, X_2) &= a e^{-(r+\bar{C})T} \left[N\left(d_2^{(X_1)}\right) - N\left(d_2^{(X_2)}\right) \right] + \\ & b S_0 e^{-(q+\bar{C})T} \left[N\left(d_1^{(X_1)}\right) - N\left(d_1^{(X_2)}\right) \right], \end{aligned}$$

where

$$d_1^{(X)} = \frac{\log(S_0/X) + (r - q + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}, \quad d_2^{(X)} = d_1^{(X)} - \sigma\sqrt{T}.$$

Li (1998) shows that the valuation of LS option is derived as the present value of an integration of the final payoff function given the lognormal density distribution of S_T, $g(S_T|S_0)$, see equation (2). The valuation of a DBLS option, a double barrier version of LS, is obtained by discounting the the same integration of a payoff function f_{LS} with conditional density distribution of S_T, given that neither barrier is breached $\min(\tau_L, \tau_U) > T$.[5] The conditional density function has the form of:

$$\begin{aligned} & \hat{g}(S_T|S_0, \min(\tau_L, \tau_U) > T) \\ = & \sum_{n=-\infty}^{\infty} \left[g\left(S_T \middle| S_0 \left(\frac{U}{L}\right)^{2n} \right) - g\left(S_T \middle| \left(\frac{U^2}{S_0}\right) \left(\frac{U}{L}\right)^{2n} \right) \left(\frac{U}{S_0}\right)^{\frac{2\lambda}{\sigma^2}} \right] \left(\frac{U}{L}\right)^{\frac{2n\lambda}{\sigma^2}}. \end{aligned} \quad (7)$$

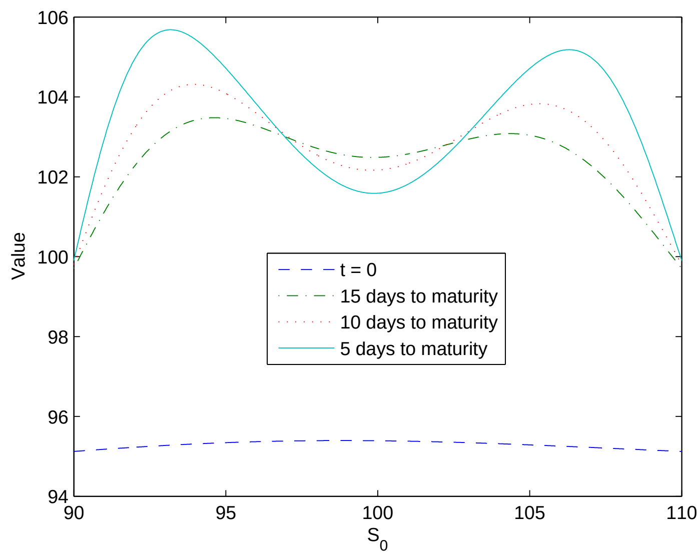
2.3 Analysis

In Figure 3 we show graphically the value of an ARBN as a function of its underlying security over time. A simple ARBN is created assuming the initial stock level $S_0 = \$100$, the same as principal. At issuance ($t = 0$), since we discount the future cash flows, the value is less than par (\$100). When the value of the underlying security is above the upper barrier ($U = \$110$) or bellow the lower barrier ($L = \$90$), the value of the ARBN is the discounted value of its face value (\$100). The closer the ARBN is to maturity (conditional on it not having broken any barrier until that point) the more pronounced

[5]This conditional density distribution is well analyzed, see for example Anderson (1960) and He et al. (1998).

the double hump shape of the value function. This shape is due to the double barriers. The higher the price of the underlying security is, the higher the return, up to a point where the price of the underlying security is close to the barrier and then the probability it will breach the barrier is higher and hence the value of the ARBN is reduced. In order to better understand this dynamics we compute the delta of the ARBN.

Figure 3: Fair value of ARBNs as a function of the underlying security price



To calculate the delta of an ARBN, we use the same decomposition approach:

$$\Delta_{ARBN}(\vec{S}_t) = \Delta_{DBLS}(\vec{S}_t, -S_0, 1, S_0, U) + \Delta_{DBLS}(\vec{S}_t, S_0, -1, L, S_0). \tag{8}$$

Where the delta for DBLS is calculated as

$$\begin{aligned}
\Delta_{DBLS}(\vec{S}_t, a, b, X_1, X_2) \quad = \quad & \sum_{n=-\infty}^{\infty} \left[\Delta_{LS}\left(S_0 \left(\frac{U}{L}\right)^{2n}, a, b, X_1, X_2 \right) \left(\frac{U}{L}\right)^{2n} - \right. \\
& \Delta_{LS}\left(\frac{U^2}{S_0} \left(\frac{U}{L}\right)^{2n}, a, b, X_1, X_2 \right) \left(\frac{U}{S_0}\right)^{\frac{2\lambda}{\sigma^2}} U^2 \left(\frac{U}{L}\right)^{2n} \\
& \left. + V_{LS}\left(\frac{U^2}{S_0} \left(\frac{U}{L}\right)^{2n}, a, b, X_1, X_2 \right) \left(\frac{U}{S_0}\right)^{\frac{2\lambda}{\sigma^2}} \left(-\frac{2\lambda}{\sigma^2 S_0} \right) \right] \left(\frac{U}{L}\right)^{\frac{2n\lambda}{\sigma^2}}.
\end{aligned}$$

The delta of the LS option used in the above formula is

$$\begin{aligned}\Delta_{LS}(S_0, a, b, X_1, X_2) \quad = \quad & be^{-(q+\bar{C})T}\left[N\left(d_1^{(X_1)}\right) - N\left(d_1^{(X_2)}\right)\right] + \\ & \frac{e^{-(q+\bar{C})T}}{\sigma\sqrt{T}}\left[\left(\frac{a}{X_1}+b\right)N\left(d_1^{(X_1)}\right) - \left(\frac{a}{X_2}+b\right)N\left(d_1^{(X_2)}\right)\right].\end{aligned}$$

Figure 4: Delta of ARBN at different times

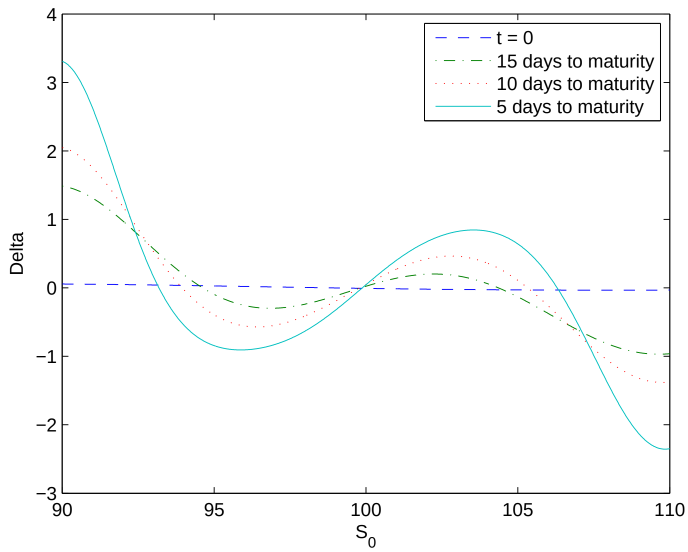
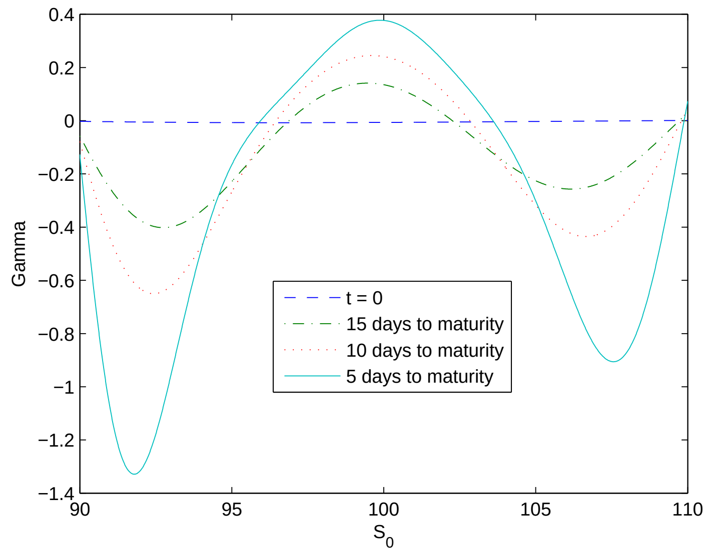


Figure 4 describes the delta of an ARBN, the sensitivity of the value of the ARBN to changes in the underlying security. At the beginning of the ARBN, the delta is very close to zero as it is far enough from maturity to not be very sensitive to the changes in the underlying security. As the ARBN gets closer to maturity, the delta is positive when the underlying security price is above the reference price but low enough to likely not exceed the upper barrier. Conversely, the delta exhibits a pattern of being negative when the underlying security price is below the reference price but high enough to likely not breach the lower barrier. The closer the price of the underlying security is to the upper or lower barrier the more the ARBN's price is sensitive to changes in the underlying security price and the.

We also derive the Gamma of an ARBN and graph it in Figure 5. As with the Delta and the value of the ARBN, the Gamma becomes sensitive the closer we get to maturity.

Figure 5: Gamma of ARBNs at different times



Close to maturity, the Gamma increases as the price gets closer to S_0. The Gamma decreases and becomes negative negative as the price moves away from S_0. Interestingly, once the price is closest to the boundaries, the Gamma increases again.

3 Valuation of Real-World ARBNs

We collect data on 279 ARBNs issued from 2006 and 2009 by the main six investment banks that issue ARBNS (Deutsche Bank, Goldman Sachs, HSBC, Lehman Brothers, Morgan Stanley and UBS). The investment banks market their ARBNs under slightly different names, such as 'Absolute Return Trigger Notes' (Goldman Saches), 'Protected Absolute Return Barrier Notes' (Morgan Stanley), and '100% Principal Protection Absolute Return Barrier Notes' (UBS) Moreover, there are also other variations in the structure of ARBNs. For example, *autocallable* ARBNs pay the principal back when the barriers are breached rather than at maturity[6], and *buffered* ARBNs provide a buffer against losses but do not protect 100% of the note's principal.

The 279 ARBNs have an aggregate face value of $4.2 billion, and are generally linked to

[6]See (Deng et al., 2011) on the general approach to valuing the autocallable feature structured products.

indices. The primary indices used are the S&P 500 Index (63% of issues) and the Russell 2000 Index (13%) (see Table 1.) Others underlying securities include the Nasdaq 100 Index, ETFs, and exchange rates. The narrow distribution of underlying securities is in line with Henderson and Pearson (2010)'s conclusion that issuers prefer using well-known underlying securities.

Table 1: Distribution of ARBN Underlying Securities

Issuers	S&P 500 Index	Russell 2000 Index	Other Indices	Non-index	Total
Deutsche Bank AG	60	15	8	19	102
Goldman Sachs	26		1	7	34
HSBC	3	4	1	2	10
Lehman Brothers	10	3	4	2	19
Morgan Stanley	30		12	5	47
UBS	47	15	3	2	67
Total	176	37	29	37	279

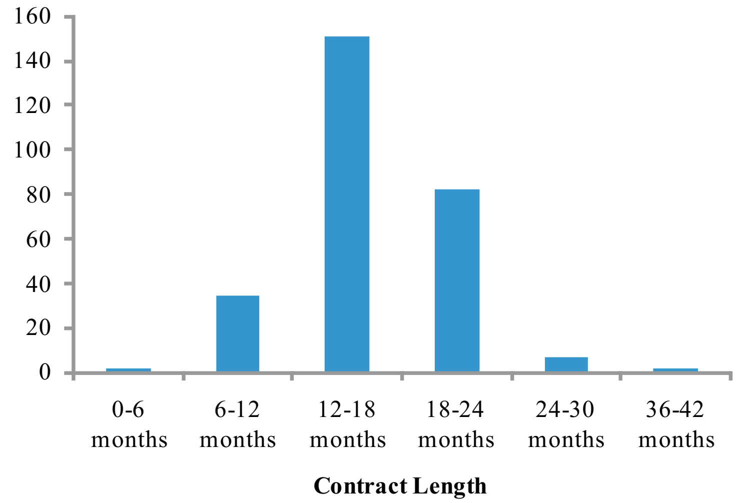
Generally speaking, ARBNs are short-term investments. Figure 6 charts the maturities of the ARBNs in our sample. The maturities range from 6 months to 3 years, but most maturities are between 12 and 18 months. The underwriting fees on these product range from 1% to 2%. Underwriting fees include a fee paid to investment companies who market the product and commissions paid to the investment advisor who sell the product to the investor. In our sample, UBS is frequently used as the marketing agent. The issuer must charge more than the underwriting fees plus the market value of the product in order to make a profit.We restrict our sample to standard ARBNs for which we can collect the necessary information, including the underlying security's implied volatility. Our final sample set contains 214 ARBNs.

On average, the fair value of an ARBN in our sample is 95.5% of the product's principal, meaning the note is issued at a 4.5% premium. In Table 2 we present the ARBNs' fair values and implied yields by investment banks. The implied yield is defined as the interest rate that makes the fair value equal to the issue price (at par). Morgan Stanley tends to have a lower average price at issuance than the other issuers. This may be due to Morgan Stanley's high credit default swap spread, which is incorporated in the valuation model.

Figures 7(a) to 7(c) plot the implied yield of each ARBN against the 1-year LIBOR rate and the issuer's 1-year bond yield equivalent.[7] As the figures show, all of the ARBNs'

[7]For simplicity, the corporate bond yield is equivalent to sum of the LIBOR rate and the issuer's CDS.

Figure 6: Contract Length Distribution



implied yields are lower than the corresponding corporate yields, and many are even lower than the risk-free rate. We find that Lehman's structured products generally have implied yields below the 1-year LIBOR rate. This indicates that Lehman used structured products including ARBNs to debt-finance its operations at sub-market rates, especially when the company's credit quality decreased sharply in 2007 and 2008.

In Table 3 we describe how many ARBNs from each issuer have matured and returned a positive return. 173 of the 214 ARBNs have matured as of December 31, 2009. Of the 173 issues, 11 defaulted (all Lehman's), 119 breached a barrier and returned the face value to investors, and 43 paid investors a positive return. Considering all the 162 ARBNs that have matured and did not default, the average return on the notes was 3.5%. For the 43 issues that paid a positive return, the average return was 13.4%.

4 Conclusion

In this paper we present a closed-form valuation of the standard type of Principal Protected Absolute Return Barrier Notes (ARBNs). There is a variety of approaches to valuing ARBNs. Our approach focuses on decomposing ARBNs into a zero-coupon bonds and linear segment options. As a by-product of the decomposition approach, we also evaluate Greeks such as delta and gamma.

Table 2: Fair Valuation of the ARBNs

Issuer	Fair Price at Issue Time	Average Implied Yield
Deutsche Bank AG	95.94%	1.31%
Goldman Sachs	95.69%	1.00%
HSBC	96.69%	1.47%
Lehman Brothers	95.06%	1.49%
Morgan Stanley	91.92%	1.11%
UBS	96.86%	1.32%
Total	95.58%	1.32%

Table 3: Actual Returns for the ARBNs

Issuers	Total Matured Products	Positive Returns	Pay Principal
Deutsche Bank AG	68	15	53
Goldman Sachs	22	7	15
HSBC	8	2	6
Lehman Brothers	11 (defaulted)	3 (defaulted)	8 (defaulted)
Morgan Stanley	16	5	11
UBS	48	14	34
Total	173	46	127

We apply our valuation method to a sample of 214 ARBNs and find that the ARBNs in our sample are issued at a 4.5% premium. This premium is lower than the premia on European reverse convertible products presented in Henderson and Pearson (2010) but is close to the premia on U.S. dollar-denominated reverse convertibles discussed in Hernández et al. (2007). We further conclude that investment banks may use ARBNs as a cheap financing tool to attract investments from unsophisticated investors.

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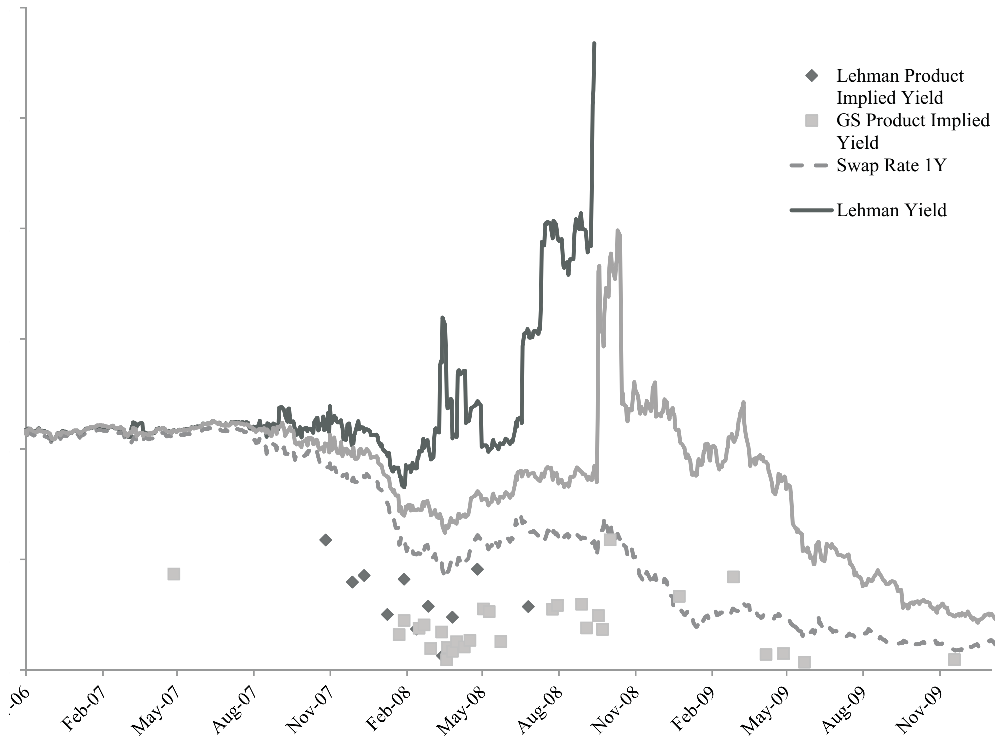
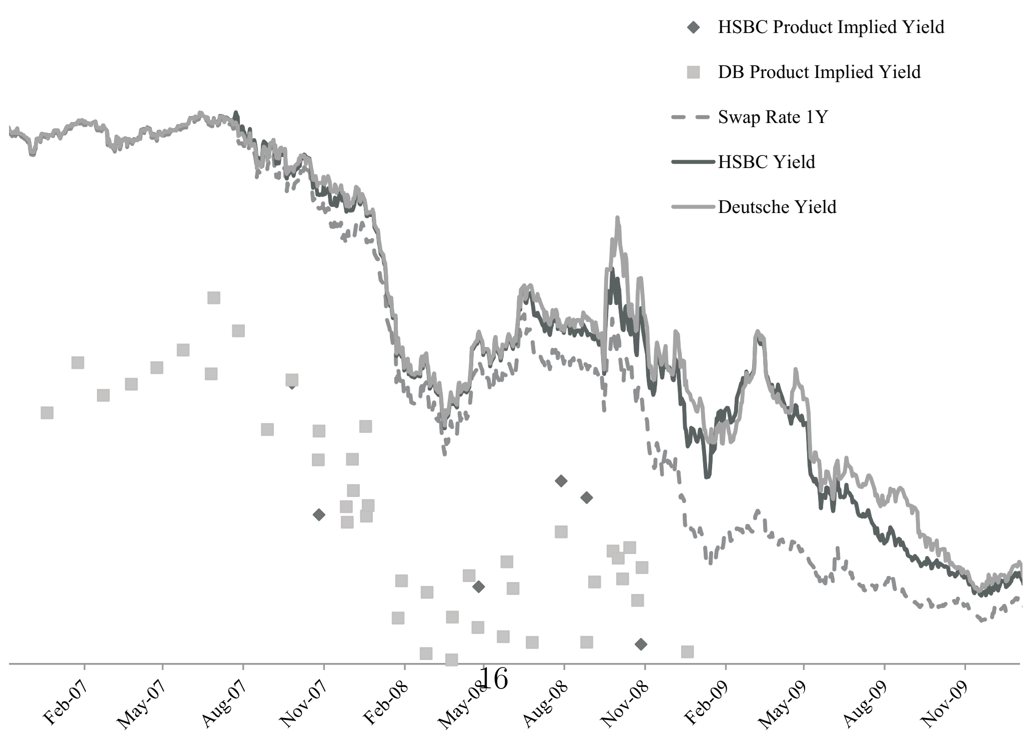


(a) Lehman Brothers and Goldman Sachs



(b) HSBC and Deutsche Bank