

Monte Carlo Simulation

Introduction

Monte Carlo simulations have been widely used in risk management and in financial planning since the early 1990s.

Monte Carlo simulation is a technique for answering statistical questions which are too complicated to answer with simple equations.¹

- A client wants to know the impact of different asset allocations on the risk that his investment portfolio will not provide for him in retirement.
- Another client has a portfolio concentrated in a low basis stock and wants to know whether she should sell some or all of the stock and diversify the after tax proceeds.
- A pension fund trustee asks What the largest loss the fund could suffer in ordinary times?

We Estimate Return Distributions

The answers to these questions will depend on the interaction between uncertain future investments returns and investment and consumption decisions.

Monte Carlo simulation combines the returns randomly drawn from probability distributions with assumptions about deposits and withdrawals, inflation, etc. to calculate changes in the value of a portfolio over time. The probability distribution of future investment returns could be supplied by finance theory, empirical research or both.

Repeating this process of randomly drawing a return from the distribution for each successive year out into the distant future allows the analyst to sketch out a path for the investment portfolio into the distant future.

Once the simulation software is set up, 10s of 1,000s of these possible future paths can be generated. By counting up the number of the paths which satisfy some condition, the analyst

can determine the probability that the retirement portfolio will run out, that the concentrated stock position will be worth more in the future after taxes if held rather than diversified, or the most the pension fund will lose in ordinary times.

An Example

Table 1 presents a stylized Monte Carlo analysis. Given our assumptions we find that 14% of the projected paths deplete the portfolio within 10 years, 59% deplete the portfolio within 20 years and 75% deplete the portfolio within 30 years.

Inputs	Expected Return	Standard Deviation	Asset Allocation
Stocks	10%	30%	70%
Bonds	5%	5%	30%
Stock / bond correlation	0.4		
Initial Portfolio	\$500,000		
Initial Withdrawal ²	\$40,000		
Results	Expected Return	Standard Deviation	
Portfolio	8.8%	21.6%	
Probability of running out of money within			
	10 years		14%
	20 years		59%
	30 years		75%

With Monte Carlo simulations we can easily test the impact of changing our assumptions on the probability of financial ruin. For example, holding everything else constant if we reduce the stock allocation to 50%, the probability of ruin at 30 years increases slightly, the probability at 20 years is unchanged, and the probability at 10 years drops significantly.

Conclusion

Monte Carlo simulation is a powerful tool for estimating the risk of investments. The simulations can be set up in Excel spreadsheets and the assumptions varied to determine the impact of alternative decisions on the client's objectives.

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¹ Versions of these questions can be answered with closed form equations but Monte Carlo simulation often makes the results more easily understood.

² We assume the annual withdrawal increases with inflation at 3%.