An Economic Analysis of Reverse Exchangeable Securities
— An Option-Pricing Approach

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Abstract

In this paper we provide economic and empirical analyses for reverse exchangeable bonds. We make a detailed survey of the $ 45 billion US dollar-denominated market for 7,426 issues of bonds issued between May 1998 and February 2007. In addition, we also develop pricing models for four types of bonds and empirically examine the profits for issuing these bonds. The results of the survey suggest significant positive profits for the issuing financial institutions. We also show that a perfect hedge can be obtained through *static* and *costless* trading strategies and find that issuing reverse exchangeables with the perfect hedging strategy has a payoff identical to the payoff of a call option.

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I. Introduction:

The development of structured products -- that is to create new securities through the combination of fixed income securities, equities and derivative securities -- has been rapidly accelerating for more than one decade. The creation, underwriting and trading of structured products have become a significant source of revenues for many investment and commercial banks.

The development of structured products is part of the financial innovation process that provides important functions in the financial market. For instance, the structured products enhanced the capital market efficiency by combining the transactions of several securities into one and thus reduced transaction costs. The development of structured products has also challenged practitioners, academicians, and regulators. For instance, some structured products may include exotic derivatives that are difficult to price and regulators are concerned that some structured products may be too complicated for unsophisticated investors to understand.[1] However, the complication of the products and regulators' concerns of investors' inability to understand the risk have not slowed down the development and marketing of such products. Instead, the trend in the market is the design of more complicated structured products (e.g. moving from *standard* options to *exotic* options) and targeting individual investors as primary customers.

In this paper, we introduce a product known as "reverse exchangeable bond" to examine how the product is structured. We especially examine how plan vanilla *standard* options were replaced

[1] For instance, the National Association of Securities Dealers expressed its concerns of unsophisticated investors' investment in structured products in its publication *Notice to Members 05-59* entitled *"Guidance Concerning the Sale of Structured Products"* (September 2005).

by more complicated *exotic* options in the design of the products. We provide detailed descriptions and analyses of the market for the $45 billion US dollar-denominated reverse exchangeable bonds issued between May 1998 and February 2007. We also examine whether and how the issuers make a profit in the primary market and how issuers can perfectly hedge the risk for issuing such bonds.

Several studies (e.g. Benet, Giannetti, and Pissaris (2006), Burth, Kraus, and Wohlwend (2001), Grünbichler and Wohlwend (2004), Stoimenov and Wilkens (2005), Szymanowska, Horst, and Veld (2004), Wilkens, Erner, and Roder (2003)) have consistently reported that these bonds were overpriced based on theoretical pricing models. In this paper, we extend the previous studies to US dollar-denominated securities making an extensive survey of its market.

The rest of the paper is organized as follows: we introduce the structure of the products in Section II. We then provide detailed analyses of the $45 billion US dollar-denominated reverse exchangeable bond market which includes 7,426 bonds issued between May 1998 and February 2007 (Section III). We further analyze these structured products and show how these reverse exchangeable bonds can be decomposed into zero coupon bonds and short positions in put options on the underlying assets (Section IV). Furthermore, we price 6,515 issues of the reverse exchangeable bonds that have complete data based on the valuation models developed in the paper and analyze the profitability of these bond issues. We find that on average issuers make a profit of 3%-6% in the $45 billion market (Section V). We further show that the hedging strategy for plain vanilla bonds and discount certificates can be *perfect* (i.e. complete risk-free) and *costless.* Moreover, we find that the prefect and costless hedging can be reached through one-time purchase of the underlying asset (i.e. no *dynamic* hedging is needed). These results are presented in Section VI of the paper. We further explore how the underlying assets of the securities are selected in Section

VII, present the realized results for issues that expired in or before April 2007 in Section VIII. We conclude the paper in Section IX.

II. Description of the Products:

A. Plain Vanilla Reverse Exchangeable Bond:

A plain vanilla *reverse exchangeable bond* is a bond issued (usually) by a financial institution which pays a relatively high fixed coupon rate on and before the maturity date, just like a typical bond. However, the "principal payment" to be made by the issuer on the maturity date is not necessarily the "face value" as in a traditional bond. Instead, the "principal payment" is a contingent claim – contingent upon the price of a pre-specified asset (to be referred as the *underlying asset*) on the maturity date of the bond. If the closing price of the underlying asset on the bond maturity date (known as the *valuation price*) is *equal to* or *higher than* the a pre-determined price (to be referred to as the *exercise price* or *strike price*), the bond investors will receive the "face value" of the bond (usually $1,000) from the bond issuers just like a traditional bond. However, if the closing price of the underlying asset is *below* the pre-determined *exercise price*, the investors will receive a fixed number of shares of the underlying asset from the bond issuers. The number of shares is pre-specified and it is usually equal to the face value of the bond divided by the exercise price.[2] Usually the exercise price is set equal to the price of the underlying asset on the bond issue date (to be referred to as the initial asset price or *initial price*). Therefore, we may use the terms *exercise* price and *initial* price interchangeably in the paper.

If we denote I_0 as the initial price and I_T as the *valuation price*, then for an initial investment in one reverse exchangeable bond, the total value that an investor will receive on the *expiration date* (known as the *redemption value* or *settlement amount*), V_T, is equal to:

[2] To be precise, in case a fraction of a share is to be delivered by the bond issuer, cash will be paid in lieu of fractional shares.

$$V_T = \begin{cases} \$1{,}000 & \text{if} \quad I_T \geq I_0 \\ I_T \dfrac{\$1{,}000}{I_0} & \text{if} \quad I_T < I_0 \end{cases} \qquad \ldots(1)$$

Alternatively, the relationship between the redemption value on a reverse exchangeable bond and the valuation price of the underlying asset can be represented in Figure 1.

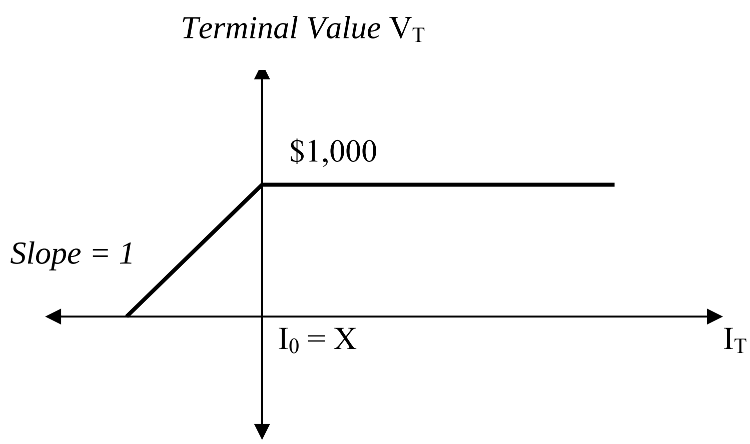

Figure 1: The terminal value of a Reverse Exchangeable Bond, V_T, as a function of the valuation price of the underlying asset, I_T.

There are at least two differences between a *convertible* bond and a *reverse exchangeable* bond. First, in the case of a convertible bond, it is the *owner* of the bond that has the right to convert the bond into the underlying assets. In a case of a *reverse* exchangeable bond, however, it is the *issuer* of the bond that has the right to deliver the underlying asset to the investors, and that is why the term "*reverse*" is used in the security. Secondly, for a *convertible* bond, the underlying *stock* that the bond investors have the right to convert to is issued by the *same* firm that issues the *bonds*. For a reverse *exchangeable* bond, however, the issuer of the underlying asset is virtually always different from the issuer of the bond. This is why the term "exchangeable" is used.[3]

When investors purchase a reverse exchangeable bond they basically engage in two transactions simultaneously: they take a *long* position in a typical fixed-rate bond and they *short* several contracts of *put* options. The *underlying asset* of the put option is the underlying asset of the reverse exchangeable, the *exercise price* of the put option is the *initial price* of the underlying asset, the *expiration date* of the put option is the maturity date of the reverse exchangeable, and the *number of contracts* written is the face value of the reverse exchangeable bond (usually $1,000) divided by the initial price of the underlying asset. The bond issuer will exercise the option by delivering the underlying asset to the bond investor when the underlying asset price on the maturity date of the bond is lower than the exercise price. The high coupon payments made by the reverse exchangeable basically include the option premium paid by the bond issuer to the investors of the reverse exchangeable bonds.

In addition to the plain vanilla reverse exchangeables, there are three other types of reverse exchangeables, they are discount certificates, *knock-in* reverse exchangeables, and *knock-out* reverse exchangeables. We will introduce each of them briefly as follows:

B. Discount Certificate:

A *discount certificate* is a special case of a plain vanilla reverse exchangeable in that a discount certificate does not make coupon payments. A discount certificate, therefore, can be viewed and priced as a *plain vanilla reverse exchangeable bond* with a coupon rate equal to zero.

[3] Most "reverse exchangeable" issuers, however, use the incorrect term "reverse convertible" when they really mean "reverse exchangeable" because the underlying assets that they have the right to deliver to investors are the stocks issued by *other* companies, rather than of *their own*. In the paper, we will use the correct term "reverse exchangeable" for such bonds.

C. Knock-In Reverse Exchangeable Bond:

A *knock-in* reverse exchangeable bond is similar to a plain vanilla reverse exchangeable except that in a *knock-in* reverse exchangeable the bond issuer has the right to exercise the option of delivering the underlying asset to bond investors *only if* the underlying asset price has dropped to a predetermined level (which is usually set *below* the initial price) anytime between the issue date and the maturity date of the bond. The predetermined level of the underlying asset price is referred to as the *knock-in level* (or *limit price*).[4] Since the knock-in level is generally set *below* the initial price, the bonds are also referred to as "*down-and-in*" reverse exchangeables. On the maturity date of the bond (t=T), for each bond the issuer will pay to investors the par ($1,000) or deliver to the investor the shares of the underlying security (known as the *redemption amount*) according to the following conditions:

$$V_T = \begin{cases} \$1{,}000 & \text{if} \quad I_T \geq I_0 \\ \$1{,}000 & \text{if} \quad I_T < I_0 \text{ and all } I_t > H,\ t \in [0, T] \\ \dfrac{\$1{,}000}{I_0} I_T & \text{if} \quad I_T < I_0 \text{ and some } I_t \leq H,\ t \in [0, T] \end{cases} \qquad \ldots(2)$$

Where $t \in [0, T]$ is the time between the issue date of the bond and the maturity date of the bond. The H in Equation (2) is the pre-specified knock-in level.

D. Knock-Out Reverse Exchangeable Bond:

A *knock-out* reverse exchangeable bond is similar to a plain vanilla reverse exchangeable except that in a *knock-out* reverse exchangeable the bond issuer loses the option of delivering the underlying asset to the investors if the underlying asset price moves above a predetermined level (which is usually set *above* the initial price) anytime between the issue date and the maturity date

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[4] Usually the knock-in level is set up as a percentage of the *initial price* (e.g. 70% of the initial price). A bond with a knock-in level of, for example, 70% of the initial price, is also referred to as having a 30% *downside protection*.

of the bond. The predetermined level of the underlying asset price is referred to as the *knock-out level* (sometimes also referred to as *limit price*). Since the knock-out level is set *above* the initial price, the bonds are also referred as "up-and-out" reverse exchangeables.[5]

The redemption amount of a knock-out reverse exchangeable bond on maturity date T is given as

$$V_T = \begin{cases} \$1{,}000 & \text{if} \quad I_T \geq I_0 \\ \$1{,}000 & \text{if} \quad I_T < I_0 \text{ and some } I_t \geq H, \ t \in [0, T] \\ \dfrac{\$1{,}000}{I_0} I_T & \text{if} \quad I_T < I_0 \text{ and all } I_t < H, \ t \in [0, T] \end{cases} \qquad \ldots(3)$$

Where H is the pre-specified knock-out level.

In Appendixes 1 and 2, we present the summary information for two reverse exchangeable bonds in our sample: one *plain vanilla* reverse exchangeable bond and one *knock-in* reverse exchangeable bond respectively.

III. The Reverse Exchangeable Bond Market:

In Table 1 we present the descriptive statistics for the four types of the products: *plain vanilla,* the *discount certificate,* the *knock-in,* and the *knock-out r*everse exchangeable bond markets. The total value issued is \$6.7 billion on 665 issues of *plain vanilla* bonds; \$9.9 billion on 2,016 issues of *discount certificates;* \$28.0 billion on 4,662 issues of *knock-in* bonds; and \$0.4 billion on 83 issues of the *knock-out* bonds. The *combined* value of reverse exchangeable bonds is about \$45 billion on 7,426 issues. The median term to maturity is close to one year for all types except for discount certificates that have a median term to maturity of 59 days. The

[5] Usually the knock-out level is set up as a percentage of the initial price (e.g. 120% of the initial price).

median issue size ranges from \$2 million to \$5 million and the median coupon rate ranges from 9% to 12% for the interest bearing types. The median barriers are set at 80% and 120% of the strike price for the *knock-in* bonds and *knock-out* bonds respectively.

In Table 2 we break down the statistics for the reverse exchangeable bond markets by the *issue year* and the *type*. The *combined* reverse exchangeable bond market consists of 7,426 bonds issued between May 1998 and February 2007. One phenomenon especially worth noting is that the market is growing extremely rapidly. In terms of numbers of new issues, the compounded annual growth was at an amazing rate of 75% over the last seven years (from 70 issues in 1999 to 3,537 issues in 2006). The compounded annual growth was even higher at an astonishing rate of 136% over the last two years (from 633 issues in 2004 to 3,537 issues in 2006). In terms of dollar amount, the growth was equally impressive. The compounded annual growth was an amazing rate of 40% over the last seven years (from \$1,715 million in 1999 to \$18,526 million in 2006), and it was even higher over the last two years at an astonishing rate of 113% (from \$4,066 million in 2004 to \$18,526 million in 2006). In addition, the composition of the bond *types* also migrates over time from bonds featured with plain vanilla options to bonds characterized with exotic options. For instance, the percentage of *plain vanilla* bonds and *discount certificates* decreases from 90% of the total issues in 1999 to less than 20% in 2006. On the other hand, the percentage of *knock-ins* increases from 10% of the total issues to 80% to the total issues during the same period.

In Table 3 we further break down the number of issues and dollar amount by the countries in which the issuing banks are located and by year. It is apparent that Netherlands and Great Britain dominate the reverse exchangeable bond market, followed by Germany and the United States. Out of a total of 7,426 issues of reverse exchangeable bonds (for a total value of \$45 billion), 2,931 issues (for a total value of $11.3 billion) were issued in Netherlands, 1,906 issues for a total of value of $17.9 billion) were issued in Great Britain, 1,117 issues (for a total value of $5.6 billion) were issued in the United States, and 879 issues (for a total value of $7.5 billion) were issued in Germany. During the earlier years, the market was dominated by banks or branches of banks located in Great Britain (Barclays, UBS, Goldman Sachs, Credit Suisse Bank, Credit Lyon, etc), and in Netherlands (ABN-AMRO, ING, RaboBank, etc). However, banks located in the US are catching up recently. The banks or branches of banks in the US issuing reverse exchangeable bonds include Societe Generale, Wachovia, Morgan Stanley, Merrill Lynch, Lehman Brothers, JP Morgan, and Bear Stearns. Their market shares represent about 20% of the market in terms of dollar amount as well as in number of issues. Although not reported in Table 3, our data indicates the top 5 banks measured by total number of issues between 1999 and February 2007 are: ABN-AMRO (31%), Commerzbank (10%), Barclays (9%), Societe Generale (8%), and Credit Lyon (6%).

We also classify the issues by the industries of the underlying securities based on the two-digit SIC codes and the results of the classification are reported in Appendix 3. As shown in the Appendix, the two industries in which the securities are most frequently used as underlying securities are Electronic and Other Electrical Equipment and Components (SIC code 36), and Industrial and Commercial Machinery and Computer Equipment (SIC code 35). Of the 7,426 issues reverse exchangeable bonds in the sample, 17% (1,244 issues) have underlying securities in the industry of Electronic and Other Electrical Equipment and Components (SIC code 36), and 14% (1,017 issues) have underlying securities in the industry of Industrial and Commercial Machinery and Computer Equipment (SIC code 35).

IV. The Pricing of Reverse Exchangeable Bonds:

A. Plain Vanilla Reverse Exchangeable Bond:

In this section we will develop the pricing model for plain vanilla reverse exchangeable bonds. Assume the face value is \$1,000, the coupon payments are C per time period, and the selling price of the bond is B_0. As we show in Appendix 4 of the paper, the payoff for an initial investment in one plain vanilla reverse exchangeable bond with a *strike price* of I_0, and a term to maturity T, is exactly the same as the payoff for holding the following three positions:

1. A long position in one zero coupon bond with face value equal to \$1,000 and maturity date same as the maturity date of the reverse exchangeable;
2. A long position in zero coupon bonds of which the face values equal the coupons payments of the reverse exchangeable and maturity dates are the reverse exchangeable coupon payment dates;
3. A short position in put options with exercise price of I_0, term to expiration of T (which is the term to maturity of the reverse exchangeable), and number of options of $\frac{\$1{,}000}{I_0}$.

Since the payoff of a plain vanilla reverse exchangeable bond is the same as the combined payoffs of the above three positions, we can price the fair value of the reverse exchangeable based on the three positions. Any selling price of the bonds above the value of the three positions is the gain to the bond issuer.

The value of *Position 1* is the price of a zero coupon bond with a face value \$1,000 and maturity date T. So it has a value of $\$1{,}000\, e^{-rT}$. The value of *Position 2* is the present value of the coupon payments of the reverse exchangeable bond, $\sum_i^n Ce^{-r\,t_i}$. The value of *Position 3* is the value of $\frac{\$1{,}000}{\mathrm{I}_0}$ shares of put options with each option having the value P[6]:

$$P = \mathrm{I}_0 e^{-rT} N(\text{-}d_2) - \mathrm{I}_0 e^{-qT} N(\text{-}d_1) \qquad \ldots(4)$$

Where r is the risk-free rate of interest, q is the dividend yield of the underlying assets, T is the term to maturity of the reverse exchangeable bond, X (≡ I_0) is the exercise price [7] and

$$d_1 = \frac{\ln\left(\frac{I_0}{I_0}\right) + \left(r - q + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = \frac{\left(r - q + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Where σ is the standard deviation of the underlying asset return. Therefore, the profit function, ∏ for the issuing firm is:

$$\begin{aligned}
\prod &= B_0 - \sum_i^n Ce^{-r\,t_i} - V_T \\
&= B_0 - \sum_i^n Ce^{-r\,t_i} - \$1{,}000e^{-r\,T} + \frac{\$1{,}000}{\mathrm{I}_0} P && \ldots(5) \\
&= B_0 - \sum_i^n Ce^{-r\,t_i} - \$1{,}000e^{-r\,T} + \frac{\$1{,}000}{\mathrm{I}_0}\left[\mathrm{I}_0 e^{-rT} N(\text{-}d_2) - \mathrm{I}_0 e^{-q\,T} N(\text{-}d_1)\right] \\
&= B_0 - \sum_i^n C_i e^{-r\,t_i} - \$1{,}000e^{-rT} + \$1{,}000\left[e^{-rT} N(\text{-}d_2) - e^{-qT} N(\text{-}d_1)\right] && \ldots(6)
\end{aligned}$$

[6] The pricing formula for this put option is a special case of the Black-Scholes general model for a put in that the exercise price, X, is the same as the initial stock price (i.e. X = I_0).

[7] Theoretically, the exercise price X should be the same as I_0, the price of the underlying asset on the issue date. For most cases this is true, but there are exceptions. For instance, in some cases the underlying assets prices on the day (or a few days) before or after the issue date are used as exercise prices. In some cases the rounded underlying assets prices on the issue date are used as the exercise prices. In the empirical data, we use the actual exercise prices taken from the final term sheets.

It is also worth noting that, although the initial price I_0 is explicitly specified in the contract of reverse exchangeable bonds, I_0 vanishes both in Equation (6) and in d_1 and d_2. The fact that the profit function for issuing reverse exchangeables *independent* of the initial price I_0 is a very important feature in the design of a reverse exchangeable because once a reverse exchangeable bond is designed, it can be issued *any time* before maturity regardless the price of the underlying stock since the issuer's profit will not be affected by the price.

B. Discount Certificate:

As we show in Appendix 5 of the paper, a discount certificate is a special case of plain vanilla reverse exchangeable in which the bond is a zero coupon bond and the number of embedded put options is one. The profit function ∏ for the issuer of a discount certificate, therefore, is a reduced form of Equation (5).[8]

$$\begin{aligned} \prod &= B_0 - V_T \\ &= B_0 - \$1{,}000e^{-rT} + \frac{\$1{,}000}{X}P \end{aligned} \qquad \ldots(7)$$

where

$$P = Xe^{-rT}N(\text{-}d_2) - I_0 e^{-qT}N(\text{-}d_1) \qquad \ldots(8)$$

and

$$d_1 = \frac{\ln\left(\frac{I_0}{X}\right) + \left(r - q + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

C. Knock-In Reverse Exchangeable Bond:

The profit function ∏ for the issuer of a knock-in reverse exchangeable, therefore, is a modified form of Equation (5) where the short positions the put options are *down-and-in* puts

[8] To use a general notation to cover all possible cases for the exercise price, we use X.

instead of standard puts. Based on Hull (2003), when the knock-in level is lower than the strike price, the price for a down-and-in put, P_{di}, can be express as:

$$P_{di} = -I_0 e^{-qT} N(-x_1) + Xe^{-rT} N\left(-x_1 + \sigma\sqrt{T}\right) + I_0 e^{-qT} \left(\frac{H}{I_0}\right)^{2\lambda} \left[N(y) - N(y_1)\right]$$
$$- Xe^{-rT} \left(\frac{H}{I_0}\right)^{2\lambda-2} \left[N\left(y - \sigma\sqrt{T}\right) - N\left(y_1 - \sigma\sqrt{T}\right)\right] \quad \ldots(9)$$

Where r is the risk-free rate of interest, q is the dividend yield of the underlying stock, T is the expiration date of the put option, X is the exercise price of the put option[9], I_0 is the underlying asset initial price, σ is the standard deviation of the underlying asset return, H is the knock-in level, and

$$x_1 = \frac{\ln\left(\frac{I_0}{H}\right)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$y_1 = \frac{\ln\left(\frac{H}{I_0}\right)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$y = \frac{\ln\left(\frac{H^2}{I_0 X}\right)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$\lambda = \frac{(r-q) + \frac{\sigma^2}{2}}{\sigma^2}$$

The profit function of the issuer of the bond at the issue date t=0 is

$$\Pi = B_0 - \sum_i^n C_i e^{-r t_i} - \$1{,}000\, e^{-rT} + \frac{\$1{,}000}{I_0} P_{di} \quad \ldots(10)$$

[9] For knock-in reverse exchangeables, the exercise price is the same as the initial stock price (i.e. $X=I_0$).

D. Knock-Out Reverse Exchangeable Bond:

The profit function ∏ for the issuer of a knock-out reverse exchangeable, therefore, is a modified form of Equation (5) where the short positions of put options are *up-and-out* puts instead of plain vanilla puts. Based on Hull (2003), when the knock-out level is higher than the strike price, the price for an up-and-out put, P_{uo}, can be written as:

$$P_{uo} = P - P_{ui} \qquad \ldots(11)$$

where

P: is the regular put premium

P_{ui}: is the premium for the up-and-in put and

$$P_{ui} = -I_0 e^{-qT}(H/I_0)^{2\lambda} N(-y) + Xe^{-rT}(H/I_0)^{2\lambda-2} N(-y+\sigma\sqrt{T}) \qquad \ldots(12)$$

Where H is the knock-out level, and

$$\lambda = \frac{(r-q)+\frac{\sigma^2}{2}}{\sigma^2}$$

The profit function of the issuer of the bond at the issue date t=0 is

$$\Pi = B_0 - \sum_i^n C_i e^{-r\,t_i} - \$1{,}000\, e^{-rT} + \frac{\$1{,}000}{I_0} P_{uo}$$

$$\Pi = B_0 - \sum_i^n C_i e^{-r\,t_i} - \$1{,}000\, e^{-rT} + \frac{\$1{,}000}{I_0}\left[P - P_{ui}\right] \qquad \ldots(13)$$

The pricing formulas in Equations (10) and (13) assume that I_t is observed *continuously*. If the decision to determine whether or not the knock-in (knock-out) level is reached is based on the daily closing price of the underlying stock (i.e. based on *discretely* monitoring) Broadie, Grasserman, and Kou (1997) provide a way of adjusting the option pricing formula: the knock-in level H will be replaced by $He^{0.5826\,\sigma\sqrt{\frac{T}{m}}}$, where m is the number of times the stock price is observed (i.e. T/m is the time interval between observations).

V. The Hedging Strategies:

One important and interesting question for the issuers of plain vanilla reverse exchangeables (and discount certificates) is if and how the issuer can perfectly hedge the risk. We will show that the issuer can *perfectly* hedge the risk by purchasing 1,000/ I_0 shares of the underlying asset on the issue date at the price I_0 for each bond to be issued.

Since the only uncertain cash flow faced by the bond issuer is the *redemption value*, -V_T (all other cash flows are known at t=0), all we need to show is how the uncertainty in cash flow –V_T can be completely eliminated when the issuers purchase $1,000/ I_0 shares of underlying assets for each bond they issue.

After the purchase of $1,000/ I_0 shares of the underlying asset, the combined value of the purchased underlying assets and the contingent payment of the reverse exchangeable bond on maturity date T, F_T, will be

$$F_T = \left(\frac{\$1{,}000}{I_0}\right) I_T - V_T \qquad \ldots(14)$$

Substitute the value of V_T in Equation (1) into Equation (14), we obtain

$$F_T = \begin{cases} \dfrac{\$1{,}000}{I_0} I_T - \$1{,}000 & \text{if } I_T \geq I_0 \\ \$0 & \text{if } I_T < I_0 \end{cases}$$

$$= \frac{\$1{,}000}{I_0} \times \begin{cases} (I_T - I_0) & \text{if } I_T \geq I_0 \\ 0 & \text{if } I_T < I_0 \end{cases}$$

$$= \frac{\$1{,}000}{I_0} Max[0, I_T - I_0] \qquad \ldots(15)$$

Equation (15) is non-negative. Therefore, the total value of the contingent payment of the reverse exchangeable, -V_T, combined with the long position in the underlying asset, $1{,}000 I_T/I_0$, produces a non-negative value F_T, as shown in Equation (15). Thus, the risk faced by reverse exchangeable issuers for possible unknown negative cash flows on day T is completely eliminated.

An alternative approach to see how the complete hedge is achieved is as follows: after purchasing $1{,}000 I_T/I_0$ shares of underlying asset for each bond, the bond issuer will hold the shares to maturity date. In case $I_T \leq I_0$ on maturity date, the bond issuer can deliver to the bond investors as specified in the contract. In case $I_T > I_0$, the bond issuer can sell the underlying assets in the stock market at the total value of $(1{,}000/I_0)\, I_T$ (which is greater than \$1,000), make the payment for the face value of \$1,000, and then keep the remaining cash of $[(1{,}000/I_0)\, I_T - 1{,}000]$.

There are several features worth noting in the hedging strategy. First, the hedging is complete (as opposed to partial) in that the potential for a unknown negative cash flow on maturity date T (as well as during the bond's entire life) is *completely* eliminated. Secondly, the hedging process requires only one-time purchase of the underlying asset on t=0. Since the hedging process does not require any subsequent rebalancing or transactions after the purchase of the underlying assets on t=0, the hedging process is simple and easy to implement. Thirdly, F_T, the total value of the contingent payment of reverse exchangeables in combination with the underlying assets, has the payoff pattern of a call option. This result is not surprising. The issuer of a reverse exchangeable bond basically takes a short position in bonds, and a long position in put options on the underlying asset. After taking another long position in the underlying asset, the total position should generate a payoff for a synthetic call.[10]

[10] This relationship can be seen easily from the put-call parity

$$C = S - Xe^{-rt} + P$$

The payoff of a call can be synthetically replicated by the combination of a long position in the underlying assets S, a short position in a zero-coupon bond Xe^{-rT}, and a long position in a put option.

The fourth feature of the hedge is that the static and perfect hedge is also *costless*. In other words, the profit function for the firm issuing reverse exchangeable bonds will not be affected by the hedging position taken by the firms. To prove this argument, we will calculate the profit function for a firm taking the hedging position and show that the profit function is identical to Equation (6) –the profit function for a firm not taking any hedging position.

The $\text{Max}\left[0, \mathrm{I_T} - \mathrm{I_0}\right]$ in Equation (15) is the payoff for a long position in a call with an exercise price of I_0. The present value of the payoff $\text{Max}\left[0, \mathrm{I_T} - \mathrm{I_0}\right]$, based on Black-Scholes model, is[11]

$$Call = \mathrm{I_0} e^{-qT} N(d_1) - \mathrm{I_0} e^{-rT} N(d_2) \qquad \ldots(16)$$

Where

$$d_1 = \frac{\ln\left(\frac{I_0}{I_0}\right) + (r - q + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

$$= \frac{(r - q + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

The cash flows for a firm taking the hedging position can be depicted as Figure 2:

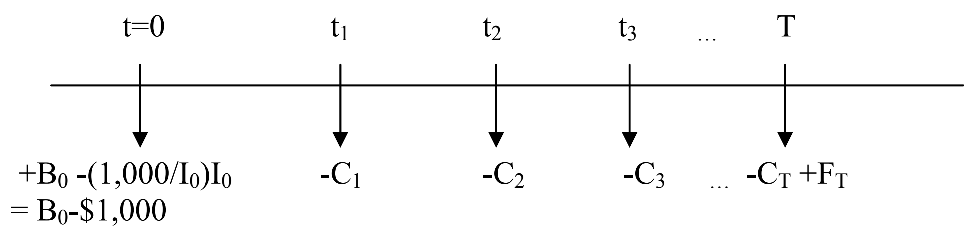

Figure 2: The cash flows for a reverse exchangeable issuer after hedging the risk of uncertain cash flows on maturity date T by taking a long position in the underlying asset on the bond issue date t=0, where F_T is the cash flow characterized by Equation (15).

[11] The pricing formula for this call is a special case of the Black-Scholes general model for a call in that the exercise price, X, is the same as the initial stock price (i.e., $X=I_0$).

The profit function based on the cash flows displayed in Figure 2 for the issuing firm at t = 0 is:

$$\Pi' = B_0 - \left(\frac{\$1,000}{I_0}\right) I_0 - \sum_i^n C_i e^{-rt_i} + PV(F_T)$$

$$= B_0 - \$1,000 - \sum_i^n C_i e^{-rt_i} + \frac{\$1,000}{I_0}\left[I_0 N(d_1) - I_0 e^{-rT} N(d_2)\right]$$

$$= B_0 - \$1,000 - \sum_i^n C_i e^{-rt_i} + \$1,000\left[N(d_1) - e^{-rT} N(d_2)\right]$$

$$= B_0 - \sum_i^n C_i e^{-rt_i} - \$1,000 e^{-rT} + \$1,000\left\{\left[N(d_1) - 1\right] - e^{-rT}\left[N(d_2) - 1\right]\right\}$$

Since

$$1-N(d_1) = N(-d_1)$$

$$1-N(d_2) = N(-d_2)$$

$$\Pi' = B_0 - \sum_i^n C_i e^{-rt_i} - \$1,000 e^{-rT} + \$1,000\left[-N(-d_1) + e^{-rT} N(-d_2)\right] \quad \ldots(17)$$

The profit function for an issuing firm taking the hedging position, ∏', in Equation (17) is identical to the profit function for an issuing firm that does not take any hedging position. Therefore, we conclude that the hedging of taking a long position in the underlying asset of a reverse exchangeable bond at t = 0 is not only *perfect*, *static* (i.e., it requires no subsequent rebalancing or any other transactions), but also *costless*.

VI. The Profitability of Reverse Exchangeable Bonds:

In this section, we examine the profits for issuing reverse exchangeables. First, we calculate the profit for each issue of bond that has complete data based on Equation (6) (for *plain vanilla* reverse exchangeables), Equation (7) (for *discount certificates*), Equation (10) (for *knock-in* reverse exchangeables), and Equation (13) (for *knock-out* reverse exchangeables) respectively. We then classify the bonds 1) by *issue year*, 2) by *term to maturity*, and 3) by *country* in which the bonds are issued. We find that issuing bonds is *profitable* for virtually all four *types* of bonds, by all *issue years*, across all the *maturities* of the certificates, and among all the *countries* in which the certificates are issued.

A. Data Description:

In order to calculate the profit, we need the following data for each bond: 1) the bond price (B_0), 2) the coupons (C) and the coupons payment dates, 3) the price of the underlying asset (I_0), 4) the cash dividends of the underlying assets and the ex-dividend dates so we can calculate the dividend yield, q[12], 5) the risk-free rate of interest, r, 6) the exercise price (X) of the options component in the certificate, 7) the volatility (σ) of the underlying asset, and 8) the term of maturity of the bond (which is also the term to expiration of the option included in the certificate), T.

The bond prices, B_0, are obtained from the final term sheets published on the web pages of issuing banks. We double check the prices and other variables in the Bloomberg Information System and several websites to ensure the accuracy of the data.[13] The prices of underlying assets are obtained from the Bloomberg System; dividend data are taken from I/B/E/S on the Bloomberg; the risk-free rates of interest are the yields on government bonds with the same maturities as the certificates.[14] The exercise prices (X) of the options, the coupons (C) paid by the bonds, the coupon payment dates (t), and the terms to maturity of the certificates (T) are all

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[12] The profits in equations (6), (7), (10), and (13) are based on continuous dividend yield. Since dividends for individual stocks are discrete, we calculate the *equivalent* continuous dividend yield for stocks that pay discrete dividends. See Appendix 6 for the details of how equivalent continuous dividend yield is calculated from discrete dividends.

[13] These websites include OnVista (Germany www.onvista.de), the Yahoo (Germany http://de.yahoo.com), ZertifikateWeb (Germany www.zertifikateweb.de), TradeJet (www.tradejet.ch), Berlim-Bremen Boerse Stock Exchange (www.berlinerboerse.de), Stuttgart Boerse Stock Exchange (www.boerse-stuttgart.de), American Stock and Options Exchange (www.amex.com), U.S. Securities and Exchange Commission (www.sec.gov), and Swiss Stock Exchange (www.swx.com).

[14] We match the maturity dates of government bonds with those of the certificates. When we cannot find a government bond that matches the term of maturity for a particular certificate, we use the linear interpolation of the yields from two government bonds that have the closest maturity dates surrounding that of the certificate.

taken from the final term sheets. The volatilities (σ) of the underlying assets are the implied volatilities obtained from the Bloomberg Information System based on the *put* options of the underlying asset. [15] For a few cases when the *implied* volatility is not available, we use the *historical* volatility calculated from the underlying securities prices in the previous 260 days.

B. *Empirical Results of the Profitability Analysis:*

In Table 4, we present the profitability for issuing reverse exchangeable by security type. The profitability is measured by the profit (∏) as a percentage of the total issuing cost (TC), i.e.

$$\text{Profitability} = \frac{\Pi}{\text{TC}} * 100\%$$

$$= \frac{B_0 - \text{TC}}{\text{TC}} * 100\% \qquad \ldots (18)$$

The results in Table 4 show that the reverse exchangeable issuers made statistically significant profits in the markets. The average profit for the 6,515 issues in the sample is a hefty 4.69% above the issuing cost. With a total market value of $45 billion, the profitability measures translate into a profit of $2.11 billion.

The profits for issuing the certificates are consistent no matter how we break down the data. We break down the profit by *issue year* (Table 5), by *terms to maturity* and by *countries* in which the bonds are issued in (Table 6). The results in these tables *consistently* indicate that the profits of issuing the bonds are statistically significantly positive. The results in Tables 4-6 suggest that issuing reverse exchangeables is a profitable business.

[15] The implied volatility calculated by the Bloomberg System is the weighted average of the implied volatilities for the three put options that have the closest at-the-money strike prices. The weights assigned to each implied volatility are linearly proportional to the "degree of near-the-moneyness" (i.e. the difference between the underlying asset price and the strike price) with the options which are closer-to-the-money receive more weight.

VII. The Selection of Underlying Assets:

In this section, we explore how underlying assets are selected. Our conjecture is, in order to enhance the marketability of the bonds, issuers are more likely to select securities highly recognized in the market. Therefore, underlying securities tend to be the stocks of large firms. In addition, stocks of large firms tend to be more liquid and their options may also be more widely held and more frequently traded and the liquidity will make hedging easier and less costly. Based on the hypothesis, we empirically examine the firm size of the underlying securities and the results are reported in Panel A of Table 7. As shown in the panel, the average market capitalization for the underlying assets ($26.1 billion) is significantly higher than the average size of the firms in the same industry both at the country level ($9.1 billion) and at the regional level ($6.5 billion). The average percentile ranking of the market capitalization for the underlying assets among all the stocks in the same industry is 84.4% at the country level and 87.1% at the regional level. The results confirm our conjecture.

Along the same line on how certificate issuers select underlying securities, we hypothesize that issuers have an incentive to use the level of dividend yield as one of the selection criteria for underlying assets. Since a higher dividend yield will lead to a higher value of the put option, P, Equation (6) suggests that a higher dividend yield implies a higher profit for reverse exchangeable issuers. We hypothesize that issuers tend to select stocks with *high* dividend yield as the underlying assets. To test the hypothesis, we compare the dividend yield of the underlying assets with the average dividend yield for all the stocks in the same industry both at the country level and at the regional level. We also calculate the percentile ranking of the dividend yield for the underlying assets among all the stocks in the same industry at both the country and the regional levels. We present the results in Panel B of Table 7. The average dividend yield for the underlying assets (2.23%) is statistically significantly higher than the average dividend yield for the stocks in the same industry at the country level (1.29%) and at the regional level (1.21%). The average percentile ranking for the dividend yield of the underlying assets among all the stocks in the same industry is 74.8% at the country level and 74.9% at the regional level. The results indicate that the dividend yields of underlying assets are significantly higher than the average dividend yield for the stocks in the same industry.

We also analyze the trend of the dividend yield and the market capitalization of the underlying securities over time and present the results in Table 8. The results in Table 8 indicate that, while the dividend yield remains virtually constant over time, the market capitalization, no matter measured by the mean, the median, or the weighted average, reaches its peak in 2001 and then gradually declines over time since 2002. The percentile ranking of the underlying assets (relative to all the firms in the same industry at the regional level) also declines recently. Furthermore, the number of securities selected as underlying assets increases monotonically over time. In addition, despite that the securities used as underlying assets increases rapidly over time, the number of bond issues *per underlying security* also increases over time (as indicated by the variable *ratio* on the last column of Table 8) due to the even greater increase in the number of reverse exchangeable bond issues recently. The analysis in table 8 indicates the reverse exchangeable bond market is expanding rapidly and the product may reach an stage of maturity soon.

VIII. The Realized Returns of Expired Cases:

We also analyze the expired reverse exchangeable bonds as of April 26, 2007. The 5,194 issues of expired bonds represent approximately 70% of the 7,426 issues of all the reverse exchangeable bonds issued between May 1998 and February 2007. In addition to the realized return for each expired reverse exchangeable bond, we also calculate, for each bond, the total return (price appreciation plus dividend) on the underlying asset as well as the total return on a benchmark index[16] over the same period as the term to maturity of the reverse exchangeable bonds and present the results in Table 9.

As shown in Table 9, over the same period as the term to maturity of the bond, the average return on the underlying assets is consistently higher than that of the benchmark index, with higher standard deviation. For instance, for the combined sample of all four types of reverse exchangeable bonds, the average return for the underlying assets is 33.90% (with a standard deviation of 188.39%) while the average return for the benchmark index is 11.38% (with a standard deviation of 31.28%). The results suggest that a typical underlying asset tend to have a higher return (and also higher risk) that its benchmark index.

The results in Table 9 also indicate that the realized return on reverse exchangeable bonds also tend to be lower than the return on underlying assets (and the index as well) with lower risk than both the underlying assets and the indices. For instance, for all four types of reverse exchangeable bonds, the average realized return is 2.87%, which is lower than the return on the underlying asset (33.9%) and the index (11.38%), while the standard deviation for the reverse exchangeable bond is 23.86%, which is also lower than the standard deviation for the underlying assets (188.39%) and the benchmark indices (31.20%).

[16] The benchmark index is the index representative of the large-capitalization stocks in the market of the underlying security. Whenever the underlying is an index, the benchmark index will be the index representative of market at the higher level of aggregation.

IX. Conclusion:

In this paper, we study the $45 billion US dollar-denominated reverse exchangeable market by examining a sample of 7,426 issues issued between May 1998 and February 2007. We develop pricing models for four types of reverse exchangeables – plain vanilla, discount certificates, knock-ins, and knock-outs – and empirically calculate the profits for issuing these bonds. We find that issuance of the certificates is a profitable business. We also show that a *perfect* hedging can be achieved for plain vanilla bonds and discount certificates through a *costless* and *static* strategy. We also find that issuing reverse exchangeables with a hedging strategy has a payoff identical to the payoff of a call option. We further find evidence that in order to enhance the profit issuers tend to select underlying securities having *higher* dividend yield and *larger* market capitalization.

This paper provides insights into the design, the payoff, the market, the pricing, the profitability, and the realized returns of expired issues of the newly created financial product.

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TABLE 1

Descriptive statistics for the reverse exchangeable bond markets. The statistics include the mean and the median values of 1) the issue size measured in millions of $, 2) the term to maturity in number of calendar days, 3) knock-in level as a percentage of the strike price, 4) knock-out level as a percentage of the strike price, 5) the coupon rates, 6) the strike price as a percentage of the underlying asset price at the time of the issue, 7) the selling price of the certificate (issue price) as a percentage of the underlying asset price at the time of the issue, 8) the estimated total value of the markets, and 9) the total number of issues of bonds.

		Total Number of Issues	Total Amount Issued ($ Mill.) [a] / Reported Cases (%)	Issue Size ($ Mill.) [b]	Maturity (# of days)	KI (%) [c]	KO (%) [c]	Coupon Rate	Strike [d]	Issue Price [d]
Plain Vanilla										
	Mean	665	6,704 (95.54%)	10.08	304	n.a.	n.a.	10.81	94.97	94.62
	Median			5.00	365	n.a.	n.a.	10.00	97.50	97.67
Discount Certificates										
	Mean	2,016	9,915 (92.06%)	4.92	135	n.a.	n.a.	0.00	94.57	89.27
	Median			5.00	49	n.a.	n.a.	0.00	93.81	89.93
Knock-In										
	Mean	4,662	27,965 (88.27%)	6.00	257	76.79	n.a.	12.14	100.10	99.91
	Median			3.00	359	80.00	n.a.	11.50	100.00	100.00
Knock-Out										
	Mean	83	409 (75.90%)	4.93	278	n.a.	120.08	9.27	97.21	96.81
	Median			2.00	362	n.a.	120.00	10.88	99.83	99.83
Total										
	Mean	7,426	45,156 (89.90%)	6.08	228	76.79	120.08	8.69	98.06	96.30
	Median			3.50	185	80.00	120.00	9.75	100.00	100.00

[a] estimated total amount issued in million dollars based on a percentage of the cases with reported issue size [b] in million dollars [c] knock-in / knock-out level as a percentage of the strike price [d] as a percentage of the underlying asset's price on the issue date

TABLE 2

Descriptive statistics for the reverse exchangeable bonds market by issue year and type. The statistics include 1) estimated total amount issued in million dollars, 2) the number of issues, and 3) the percentage of issues per type per year.

	Plain Vanilla			Discount Certificates			Knock-In			Knock-Out			Total		
Year	Amount [a]	Issues	%	Amount [a]	Issues	%	Amount [a]	Issues	%	Amount [a]	Issues	%	Amount [a]	Issues	%
1998	195	3	60.0	4	2	40.0	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	199	5	100.0
1999	504	15	21.4	939	48	68.6	129	7	10.0	n.a.	n.a.	n.a.	1,715	70	100.0
2000	709	41	23.2	1,012	92	52.0	226	23	13.0	41.53	21	11.9	2,246	177	100.0
2001	1,346	55	26.4	1,751	130	62.5	156	21	10.1	2.99	2	1.0	3,410	208	100.0
2002	377	30	10.3	1,489	246	84.3	82	16	5.5	n.a.	n.a.	n.a.	1,980	292	100.0
2003	1,147	66	13.6	1,809	371	76.3	615	48	9.9	n.a.	1	0.2	3,557	486	100.0
2004	837	103	16.3	1,226	249	39.3	2,016	279	44.1	6	2	0.3	4,066	633	100.0
2005	611	117	9.4	939	289	23.3	5,849	824	66.4	39	11	0.9	7,404	1,241	100.0
2006	1,053	203	5.7	1,493	503	14.2	15,851	2,797	79.1	208	34	1.0	18,526	3,537	100.0
2007 [b]	38	34	4.4	239	86	11.1	2,899	645	83.0	55	12	1.5	3,159	777	100.0
Total	6,704	665	9.0	9,915	2,016	27.2	27,965	4,662	62.8	409	83	1.1	45,156	7,426	100.0

[a] estimated total amount issued in million dollars based on a percentage of the cases with reported issue size [b] bonds issued on or before February 20, 2007

TABLE 3

Total dollar amount (in millions of US Dollars) and number of issues of reverse exchangeable bonds by year and by country in which the issuing banks are located and in which more than 100 bonds were issued. The data covers from 1998 to February 2007.

Panel A: Total Dollar Amount [a]

	Switzerland	Germany	Great Britain	Luxembourg	Netherlands	United States	Total
1998	n.a.	4	165	n.a.	n.a.	n.a.	199
1999	242	49	1,458	98	10	n.a.	1,715
2000	381	276	1,450	122	117	n.a.	2,246
2001	263	682	2,443	64	281	112	3,410
2002	114	215	1,361	15	720	32	1,979
2003	74	466	935	29	1,936	203	3,557
2004	40	78	1,947	200	1,349	408	4,066
2005	5	1,428	3,192	177	1,730	832	7,404
2006	45	4,367	6,159	431	4,451	3,461	18,526
2007 [b]	84	848	671	17	706	441	3,159
Total	1,253	7,529	17,875	1,128	11,330	5,608	45,156

Panel B: Issues

	Switzerland	Germany	Great Britain	Luxembourg	Netherlands	United States	Total
1998	n.a.	2	2	n.a.	n.a.	n.a.	5
1999	5	8	46	7	1	n.a.	70
2000	21	34	96	11	13	n.a.	177
2001	16	63	95	15	10	8	208
2002	7	30	66	3	176	8	292
2003	5	55	53	7	342	20	486
2004	3	18	145	55	307	88	633
2005	8	192	339	39	466	183	1,241
2006	24	425	882	51	1,362	646	3,537
2007 [b]	28	52	182	6	253	164	777
Total	117	879	1,906	194	2,931	1,117	7,426

[a] Estimated total dollar amount in millions of US Dollars based on a percentage of the cases with reported issue size [b] bonds issued as of February 20, 2007

TABLE 4

The number of issues, average term to maturity (in years), standard deviation of the underlying asset return, equivalent dividend yield, and profitability measured by the profit (∏) as a percentage of the total issuing cost for the reverse exchangeable bonds. The p-value tests the probability that the profitability is equal to zero.

Type	Statistic	Total Number of Issues	Maturity (Years)	Volatility	Equivalent Dividend Yield	Profitability in Percentage	p-value
Plain Vanilla							
	Mean	580	0.87	37.53	1.74	4.69	< 0.001
	Median		1.00	34.76	0.93	2.98	
Discount Certificates							
	Mean	1,922	0.35	41.93	1.14	3.25	< 0.001
	Median		0.13	38.10	0.00	1.32	
Knock-In							
	Mean	3,944	0.71	40.24	1.49	5.40	< 0.001
	Median		1.00	39.31	0.56	4.52	
Knock-Out							
	Mean	69	0.77	48.84	1.46	4.29	< 0.001
	Median		1.00	42.96	0.30	3.85	
Total							
	Mean	6,515	0.62	40.59	1.41	4.69	< 0.001
	Median		0.51	38.61	0.16	3.42	

TABLE 5

The number of issues, average term to maturity (in years), standard deviation of the underlying asset return, equivalent dividend yield, and profitability measured by the profit (∏) as a percentage of the total issuing cost for the reverse exchangeable bonds by issue year. The p-value tests the probability that the profitability is equal to zero.

Panel A: By Issue Year

Issue Year	Statistic	Total Number of Issues	Maturity (Years)	Volatility	Equivalent Dividend Yield	Profitability in Percentage	p-value
	Mean	5	1.59	36.82	1.73	3.80	< 0.001
1998	Median		1.03	37.77	1.40	2.16	
	Mean	68	1.02	49.63	1.21	6.69	< 0.001
1999	Median		1.00	44.54	0.39	5.19	
	Mean	171	0.95	60.97	1.03	7.55	< 0.001
2000	Median		1.01	56.63	0.16	6.03	
	Mean	197	0.95	61.92	0.85	9.12	< 0.001
2001	Median		1.00	59.07	0.23	6.23	
	Mean	288	0.55	50.53	1.17	3.48	< 0.001
2002	Median		0.24	48.62	0.00	1.43	
	Mean	469	0.46	40.91	1.03	3.35	< 0.001
2003	Median		0.17	37.52	0.00	0.97	
	Mean	606	0.67	37.58	1.43	4.70	< 0.001
2004	Median		0.97	35.37	0.00	2.79	
	Mean	1,135	0.69	35.87	1.67	4.87	< 0.001
2005	Median		0.99	34.67	0.67	3.74	
	Mean	2,964	0.58	39.66	1.55	4.59	< 0.001
2006	Median		0.50	39.10	0.18	3.77	
	Mean	612	0.52	38.39	0.97	3.98	< 0.001
2007 [a]	Median		0.48	37.60	0.00	3.29	

[a] bonds issued on or before February 20, 2007

TABLE 6

The number of issues, average term to maturity (in years), standard deviation of the underlying asset return, equivalent dividend yield, and profitability measured by the profit (∏) as a percentage of the total issuing cost for the reverse exchangeable bonds by maturity and country of the issuing bank. The p-value tests the probability that the profitability is equal to zero.

Panel A: By Maturity

Maturity	Statistic	Total Number of Issues	Maturity (Years)	Volatility	Equivalent Dividend Yield	Profitability in Percentage	p-value
	Mean	5,095	0.48	40.54	1.41	4.16	< 0.001
T ≤ 1	Median		0.27	39.05	0.00	3.07	
	Mean	1,363	1.06	41.12	1.38	6.33	< 0.001
1 < T ≤ 2	Median		1.02	37.08	0.78	4.89	
	Mean	61	2.47	32.85	2.41	12.59	< 0.001
T > 2	Median		2.04	31.27	2.18	5.20	

Panel B: By Country of the Issuing Bank

Country	Total Number of Issues	Maturity (Years)	Volatility	Equivalent Dividend Yield	Profitability in Percentage	p-value
Switzerland	95	0.77	45.67	1.42	5.50	< 0.001
Germany	822	0.62	41.95	1.57	4.75	< 0.001
Great Britain	1,635	0.85	40.33	1.46	5.74	< 0.001
Luxembourg	181	0.81	39.32	1.69	5.03	< 0.001
Netherlands	2,719	0.36	39.43	1.46	3.06	< 0.001
United States	864	0.90	43.43	0.98	7.61	< 0.001

TABLE 7

In Panel A we compare the market capitalization for all 826 underlying securities with the average market capitalization for all the firms in the same industry at the country level as well as the regional level. We also calculate the average ranking in market capitalization of underlying assets against all the firms in the same industry at the country level as well as at the regional level. In Panel B we compare the dividend yield for all 826 underlying securities with the average dividend yield for all the firms in the same industry at the country level as well as the regional level. We also calculate the average ranking in dividend yield of underlying assets against all the firms in the same industry at the country level as well as at the regional level. Based on the underlying securities' characteristics as of April 25, 2007.

	Underlying Asset	Country	Region
Panel A			
Market Capitalization (€ Million)	26,130.37	9,135.23	6,494.91
p-value [a]		< 0.001	< 0.001
Percentile Ranking [b]		84.4	87.1
p-value [c]		< 0.001	< 0.001
Panel B			
Average Dividend Yield (%)	2.23	1.29	1.21
p-value [d]		< 0.001	< 0.001
Percentile Ranking [b]		74.8	74.9
p-value [c]		< 0.001	< 0.001

[a] The probability that the average difference between the underlying asset's market capitalization and the average market capitalization for all the firms in the same industry to be zero.

[b] The formula used to compute the percentile ranking is the following:

$$\text{Percentile Ranking} = \left[\frac{\frac{N - \text{Absolute Rank} + 1}{N} + \frac{N - \text{Absolute Rank}}{N}}{2} \right]$$

[c] The probability that the percentile ranking is indifferent from 50%.

[d] The probability that the average difference between the underlying asset's dividend yield and the average dividend yield for all the firms in the same industry to be zero.

TABLE 8

The trend for the dividend yield and market capitalization of underlying securities over time. The statistics include: 1) the mean, the median, the weighted average, and the percentile ranking of a) the dividend yield, and b) the market capitalization measured in millions of $, 2) the total number of issues of bonds, 3) the total number of securities used as underlying assets, and 4) the ratio of total number of issues to the number of unique underlying securities. The data is calculated based on the underlying securities' characteristics as of April 25, 2007.

		Dividend Yield				Market Capitalization				Frequency	
Year	Issues	Mean	Median	Weighted Average [a]	Percentile Ranking [b]	Mean ($ Million)	Median ($ Million)	Weighted Average ($ Million)[a]	Percentile Ranking [b]	Number of Securities [c]	Ratio [d]
1998	5	2.01	1.8	3.17	79	147,159	153,331	125,950	96	5	**1**
1999	70	1.62	1.16	1.61	82	98,410	88,373	96,773	94	37	1.89
2000	177	1.39	0.99	1.26	88	90,667	78,405	81,235	95	60	2.95
2001	208	1.24	0.84	1.48	86	101,160	77,299	118,549	95	73	2.85
2002	292	1.86	1.45	1.98	82	110,497	74,081	135,761	96	92	3.17
2003	486	1.75	1.32	1.56	81	98,891	56,768	98,636	96	122	3.98
2004	633	1.56	1.14	1.60	84	86,367	55,034	100,144	93	189	3.35
2005	1,241	2.04	0.93	1.66	80	70,555	43,792	88,960	92	307	4.04
2006	3,537	1.73	0.76	1.67	78	46,038	24,525	55,720	89	537	6.59
Total	7,426	1.68	0.84	1.64	80	61,611	29,885	80,271	91	799	9.29

[a] weighted by the total amount issued [b] average ranking of underlying securities against all the firms in the same industry at the regional level [c] number of securities used as underlying assets [d] ratio of the number of bond issues to the number of securities used as underlying assets.

TABLE 9

Realized return for the expired cases by type. The statistics include the mean, the median, and number of observations of 1) the annualized price appreciation, and 2) the annualized total return for the certificates, underlying security, and the index comprehensive of the market of the underlying security.

		Annualized Total Return					
Security Type	Statistic	REX	Underlying		Index		
Plain Vanilla							
	Mean	1.75	15.40	a	8.40	b,c	
	St. Dev.	18.08	59.40		22.29		
	n						538
Discount Certificates							
	Mean	-0.19	45.76	a	9.62	b,c	
	St. Dev.	29.24	290.07		42.97		
	n						1,904
Knock-In							
	Mean	5.46	29.93	a	13.45	b,c	
	St. Dev.	19.90	83.53		20.94		
	n						2,692
Knock-Out							
	Mean	-6.44	1.42		1.56		
	St. Dev.	27.99	76.88		16.40		
	n						60
Total							
	Mean	2.87	33.90	a	11.38	b,c	
	St. Dev.	23.86	188.39		31.28		
	n						5,194

[a] the average difference of the underlying asset's return and the bond's return is equal to zero and significant at the 0.01 level [b] the average difference of the index's return and the bond's return is equal to zero and significant at the 0.01 level [c] the average difference of the index's return and the underlying asset's return is equal to zero and significant at the 0.01 level

Appendix 1: Example of a Plain Vanilla Reverse Exchangeable Bond

ABN AMRO Bank N.V.
MEDIUM-TERM NOTES, SERIES A
Senior Fixed Rate Notes
10.50% Reverse Exchangeable^{SM} Securities due August 18, 2005
linked to common stock of Motorola, Inc.

Securities:	10.50% Reverse Exchangeable Securities due August 18, 2005.
Underlying Shares:	Common stock, par value $3.00 per share of Motorola, Inc.
Interest Rate:	10.50% per annum, payable semi-annually in arrears on February 18, 2005 and August 18, 2005.
Issue Price:	100%
Issue (Settlement) Date:	August 18, 2004
Maturity Date:	August 18, 2005
Initial Price:	$14.21
Stock Redemption Amount:	70.37 Underlying Shares for each $1,000 principal amount of the Securities, which is equal to $1,000 divided by the initial price.
Determination Date:	The third trading day prior to the maturity date.
Payment at maturity:	The payment at maturity is based on the closing price of the Underlying Shares on the determination date. • If the closing price per Underlying Share on the determination date is at or above the initial price, we will pay the principal amount of each Security in cash. • If the closing price per Underlying Share on the determination date is below the initial price, we will deliver to you, in exchange for each $1,000 principal amount of the Securities, a number of Underlying Shares equal to the stock redemption amount. You will receive cash in lieu of fractional shares.
No Affiliation with Motorola, Inc.:	Motorola, Inc., which we refer to as Motorola, is not an affiliate of ours and is not involved with this offering in any way. The obligations represented by the Securities are our obligations, not those of Motorola. Investing in the Securities is not equivalent to investing in Motorola common stock.
Listing:	We do not intend to list the Securities on any securities exchange.

Appendix 2: Example of a Knock-In Reverse Exchangeable Bond

ABN AMRO Bank N.V.
MEDIUM-TERM NOTES, SERIES A
Senior Fixed Rate Notes
10.50% Knock-In Reverse Exchangeable[SM] Securities due August 18, 2005
linked to common stock of Circuit City Stores, Inc.

Securities:	10.00% Knock-in Reverse Exchangeable Securities due August 18, 2005.
Underlying Shares:	Common stock, par value $0.50 per share of Circuit City Stores, Inc.
Interest Rate:	10.00% per annum, payable semi-annually in arrears on February 18, 2005 and August 18, 2005.
Issue Price:	100%
Issue (Settlement) Date:	August 18, 2004
Maturity Date:	August 18, 2005
Initial Price:	$12.36
Knock-in Level:	$8.65, which is 70% of the initial price.
Stock Redemption Amount:	80.90 Underlying Shares for each $1,000 principal amount of the Securities, which is equal to $1,000 divided by the initial price.
Determination Date:	The third trading day prior to the maturity date.

Payment at maturity: The payment at maturity is based on the performance of the Underlying Shares on the determination date.

- If the market price of the Underlying Shares on the primary U.S. exchange or market for the Underlying Shares has not fallen to or below the knock-in level on any trading day from but not including the trade date to and including the determination date, we will pay you the principal amount of each Security in cash.
- If the market price of the Underlying Shares on the primary U.S. exchange or market for the Underlying Shares falls to or below the knockin level on any trading day from but not including the trade date to and including the determination date:
  — we will deliver to you a number of Underlying Shares equal to the stock redemption amount, in the event that the closing price of the Underlying Shares on the determination date is below the initial price; or
  — we will pay you the principal amount of each Security in cash, in the event that the closing price of the Underlying Shares on the determination date is at or above the initial price.
- You will receive cash in lieu of fractional shares.

No Affiliation with Circuit City Stores, Inc.:	Circuit City Stores, Inc., which we refer to as Circuit City, is not an affiliate of ours and is not involved with this offering in any way. The obligations represented by the Securities are our obligations, not those of Circuit City. Investing in the Securities is not equivalent to investing in Circuit City common stock.
Listing:	We do not intend to list the Securities on any securities exchange.

Appendix 3: Distribution of Issues Classified by Two-Digit SIC Codes of Underlying Securities

2- Digit SIC	Industry	Obs.		Frequency
	Mining			
10	Metal Mining	374		
12	Coal Mining	85		
13	Oil and Gas Extraction	563		
	Others	33	1,055	14.21%
	Construction			
15	Building Construction-General Contractors and Operative Builders	37	37	0.50%
	Manufacturing			
20	Food and Kindred Products	58		
21	Tobacco Products	89		
26	Paper and Allied Products	35		
28	Chemicals and Allied Products	526		
29	Petroleum Refining and Related Industries	187		
30	Rubber and Miscellaneous Plastics Products	31		
32	Stone, Clay, Glass, and Concrete Products	67		
33	Primary Metal Industries	406		
35	Industrial and Commercial Machinery and Computer Equipment	1,017		
36	Electronic and Other Electrical Equipment and Components	1,244		
37	Transportation Equipment	144		
38	Measuring, Analyzing, and Controlling Instruments	56		
	Others	71	3,931	52.94%
	Transp., Communications, Electric, Gas, and Sanitary Serv.			
45	Transportation by Air	111		
48	Communications	179		
49	Electric, Gas, and Sanitary Services	42		
	Others	32	364	4.90%
	Wholesale Trade and Retail Trade			
53	General Merchandise Stores	112		
54	Food Stores	30		
56	Apparel and Accessory Stores	56		
58	Eating and Drinking Places	59		
59	Miscellaneous Retail	30		
	Others	92	379	5.10%
	Finance, Insurance, and Real Estate			
60	Depository Institutions	525		
62	Sec. and Commodity Brokers, Dealers, Exchanges, and Services	159		
63	Insurance Carriers	93		
67	Holding And Other Investment Offices	253		
	Others	46	1,076	14.49%
	Services			
70	Hotels, Rooming Houses, Camps, and Other Lodging Places	40		
73	Business Services	484		
78	Motion Pictures	30		
	Others	30	584	7.86%
	Total		7,426	100.00%

Appendix 4

In this Appendix, we will show that the payoff of a plain vanilla reverse exchangeable bond is the same as the combined payoffs of the following three positions:

1. A long position in one zero coupon bond with face value equal to $1,000 and maturity date same as the maturity date of the reverse exchangeable;
2. A long position in zero coupon bonds of which the face values equal the coupons payments of the reverse exchangeable and maturity dates are the reverse exchangeable coupon payment dates;
3. A short position in put options with exercise price of I_0, term to expiration of T (which is the term to maturity of the reverse exchangeable), and number of options of $\frac{\$1{,}000}{I_0}$.

The redemption value, from Equation (1), for holding one *plain* v*anilla* reverse exchangeable bond, V_T, is:

$$V_T = \begin{cases} \$1{,}000 & \text{if} \quad I_T \geq I_0 \\ I_T \dfrac{\$1{,}000}{I_0} & \text{if} \quad I_T < I_0 \end{cases}$$

$$= \text{Min}\left[\$1{,}000, \$1{,}000 \frac{I_T}{I_0}\right]$$

$$= \$1{,}000 \ \text{Min}\left[1, \frac{I_T}{I_0}\right]$$

$$= \$1{,}000 + \$1{,}000 \ \text{Min}\left[0, \frac{I_T}{I_0} - 1\right]$$

$$= \$1{,}000 + \frac{\$1{,}000}{I_0} \ \text{Min}\left[0, I_T - I_0\right] \qquad \text{…(A4-1)}$$

The $\text{Min}\left[0, \text{I}_T - \text{I}_0\right]$ in Equation (A4-1) is the payoff for a short position for a put option with an exercise price of I_0. The negative value of V_T is

$$-\text{V}_\text{T} = -\$1{,}000 - \frac{\$1{,}000}{\text{I}_0}\text{Min}\,[0, \text{I}_\text{T} - \text{I}_0]$$

$$= -\$1{,}000 + \frac{\$1{,}000}{\text{I}_0}\text{Max}\,[0, \text{I}_0 - \text{I}_\text{T}] \qquad \ldots\text{(A4-2)}$$

The $\text{Max}\left[0, \text{I}_0 - \text{I}_T\right]$ in Equation (A4-2) is the payoff for a long position in a put with an exercise price of I_0.

The long positions in the zero coupon bonds will generate the payoff *$1,000* on maturity date T *plus* the coupons on the coupon payment dates. The payoff *$1,000* in Equation (A4-1) can be duplicated by taking a *long* position in a zero coupon bond with the face value equal to $1,000 and maturity T. The payoff $\text{Min}\left[0, \text{I}_\text{T} - \text{I}_0\right]$ in Equation (A4-1) is the payoff of a short position for a put on the underlying asset with an exercise price I_0. So the payoff for investing in one plain vanilla reverse exchangeable bond is the same as the three positions given at the beginning of the Appendix.

Appendix 5

In this Appendix, we will show that the payoff of a discount certificate is the same as the combined payoffs of the following two positions:

1. A long position in one zero coupon bond with face value equal to the exercise price, X and maturity date same as the maturity date of the discount certificate;
2. A short position in put options with exercise price of X, term to expiration of T (which is the term to maturity of the discount certificate), and a number of options equal to one.

The redemption value for holding one *discount certificate*, V_T at t=T, can be expressed as:

$$V_T = \begin{cases} X & \text{if } I_T \geq X \\ I_T & \text{if } I_T < X \end{cases}$$

$$= \text{Min}\left[X, I_T\right]$$

$$= X + \text{Min}\left[0, I_T - X\right] \qquad \ldots\text{(A5-1)}$$

The $\text{Min}\left[0, I_T - X\right]$ in Equation (A5-1) is the payoff for a short position in a put with an exercise price of X. The negative value of V_T is

$$-V_T = -X - \text{Min}\,[0, I_T - X]$$

$$= -X + \text{Max}\,[0, X - I_T] \qquad \ldots\text{(A5-2)}$$

The $\text{Max}\left[0, X - I_T\right]$ in Equation (A5-2) is the payoff for a long position in a put with an exercise price of X.

Comparing Equation (A5-2) to Equation (A4-2), we find that they are different only by a scale of $\frac{\$1{,}000}{I_0}$ when X = I_0, which is the number of contracts of the put option. Based on the results, we can conclude that a discount certificate is a special case of a plain vanilla reverse exchangeable bond when a plain vanilla reverse exchangeable bond is a zero coupon bond and when the number of contracts of the embedded put options is one.

The payoff *$1,000* in Equation (A5-1) can be duplicated by taking a *long* position in a zero coupon bond with the face value equal to X and maturity T. The payoff $\text{Min}\left[0, \text{I}_T - X\right]$ in Equation (A5-1) is the payoff of a short position for a put on the underlying asset with an exercise price X. So the payoff for investing in one discount certificate is the same as the two positions given at the beginning of the Appendix.

Appendix 6

In this Appendix, we present the approach we use calculating the equivalent continuous dividend yield for stocks that pay discrete dividends. For an underlying asset which is an individual stock with a price I_0 at t=0 (the issue date) and which pays n dividends during a time period T with cash dividend D_i being paid at time t_i, the equivalent dividend yield q will be such that

$$S_0 - \sum_{i=1}^{n} D_i e^{-r\,t_i'} = S_0 e^{-q\,T}$$

$$e^{-q\,T} = \frac{S_0 - \sum_{i=1}^{n} D_i e^{-r\,t_i'}}{S_0} = 1 - \frac{\sum_{i=1}^{n} D_i e^{-r\,t_i'}}{S_0}$$

$$-qT = \ln\left[1 - \frac{\sum_{i=1}^{n} D_i e^{-r\,t_i'}}{S_0}\right]$$

$$q = -\frac{\ln\left[1 - \frac{\sum_{i=1}^{4} D_i e^{-r\,t_i'}}{S_0}\right]}{T}$$