

## A Note On Stock Trading Models

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Damage awards in securities class action lawsuits depend on the total number of damaged shares and on the damage per share.

Damaged shares are shares bought when the stock price was artificially high due to material omissions or misrepresentations and held until the mispricing is reduced or eliminated.

It is difficult to determine the exact number of damaged shares because some shares purchased at artificially high prices are sold while prices are still artificially high; these shares are “re-traded.”

The earliest and simplest trading model, the Proportional Trader Model (“PTM”), was developed by plaintiffs’ experts. The PTM assumes that each share is equally likely to trade on any given day. Other experts have developed two classes of models—Multiple Trader Models (“MTMs”) and Accelerated Trader Models (“ATMs”).

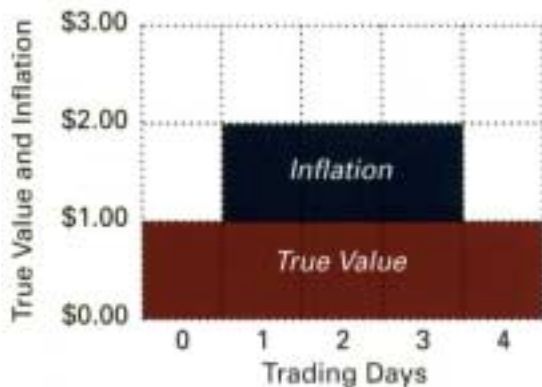
MTMs allocate trading and shareholdings to at least two types of investors and apply the PTM to each type. ATMs assume that shares that have already traded during a class period are more likely to trade than shares that have not yet traded. The amount of re-trading estimated by MTMs and ATMs is controlled by the choice of a few critical parameter values.

Published empirical research does not conclusively support one model type or one set of parameter assumptions.

Figure 1 illustrates a simple class action lawsuit covering a period of 3 days during which the share price was \$1.00 above its true value because of material misrepresentations. The difference between the market price and the true value of a share is commonly referred to as “inflation.” There were 100,000 shares available to trade and 10,000 shares traded each day.

If none of the shares bought during the class period are re-traded until after the class period, aggregate damages equal \$30,000. Investors paid \$1.00 per share too much for the first day’s 10,000 shares purchased, the second day’s 10,000 shares purchased and the third day’s 10,000 shares purchased—and lost \$1.00 on the fourth day when the fraud was disclosed. The \$30,000 estimate derived by assuming no shares bought during the class period are resold during the class period is the upper bound on damages.

Figure 1. Stock price inflated by misrepresentations





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At the other extreme, if all 10,000 shares bought each day are re-traded the next day, aggregate damages are only \$10,000. Investors who paid too much for the first day's 10,000 shares and the second day's 10,000 shares would not be damaged since they sold their purchases on the next day at the same inflation. Only purchasers of the third day's 10,000 shares are damaged, since inflation declined from \$1.00 to \$0 per share before they could sell their purchases. The \$10,000 estimate derived by assuming all shares bought during the class period are resold the next day is the lower bound on damages.

### It's the Re-trading That Matters

As our simple example illustrates, the greater the amount of re-trading assumed, the lower the resulting damage estimate, i.e., from \$30,000 for a 0% re-trading assumption to \$10,000 for a 100% re-trading assumption.

### Proportional Trader Model

The Proportional Trader Model ("PTM") assumes that each share available to trade is equally likely to trade. Thus, shares which trade each day are drawn from those which have already traded during the class period ("Traded Shares") and those which have not yet traded ("Untraded Shares") in proportion to the relative size of these two groups.

The PTM estimate of damaged shares increases each day during the class period by the number of shares bought by public investors multiplied by the proportion of the shares available to trade that are Untraded Shares.

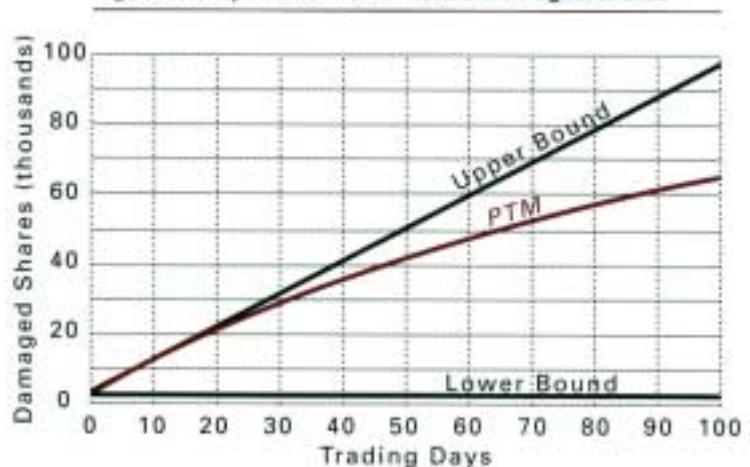
Table 1 presents the PTM analysis of our simple class action lawsuit. At the start of the class period, all of the shares available to trade are Untraded Shares and so all 10,000 shares traded the first day are damaged. On the second day, 90% of the shares of the shares available to trade are Untraded Shares and so 90% of the shares traded that day, or 9,100 shares are damaged. On the last day of the class period only 8,110 shares are damaged. 27,100 shares of the 30,000 shares traded during the class period are damaged.

The other 2,900 shares were bought and resold while the inflation was still \$1.00; therefore they were not damaged. Figure 2 illustrates the PTM's estimate of the number of damaged shares over time assuming that there are 100,000 shares available to trade and that 1,000 shares are traded

Table 1. Proportional Trader Model

Shares available to trade: 100,000/Daily Shares Traded: 10,000				
Day	Untraded Shares Available to Trade	Daily Traded Shares	Damaged Shares	Retraded Shares
	A	B	C=A*B	10,000-C
1	100.0%	10,000	10,000	0
2	90.0%	10,000	9,000	1,000
3	81.0%	10,000	8,100	1,900
Totals		30,000	27,100	2,900

Figure 2. Proportional Trader Model Damaged Shares





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each day.

### Multiple Trader Models

Multiple Trader Models (“MTMs”) assume that there are at least two types of investors, each type with a different propensity to trade. Shares available to trade and daily trading volume are allocated among these types of traders and the PTM is applied to each type separately. The separate PTM results are then added together to arrive at total estimated damaged shares.

MTMs allow for more re-trading and therefore may produce lower damage estimates than the PTM.

Table 2 presents an MTM analysis for our example lawsuit assuming two types of traders: a high activity type, which holds 20% of the shares available to trade and does 80% of the daily trading, and a low activity type, which holds 80% of the shares available to trade and does 20% of the daily trading.

Consider, first, the active traders. They hold 20,000 shares and trade 8,000 shares per day. At the start of the class period, all of the shares available to trade are Untraded Shares and so all 8,000 shares traded the first day are damaged. On the second day, 60% of the shares available to trade are Untraded Shares and so 60% of the shares traded that day, or 4,800 shares, are damaged. On the last day of the class period 2,880 shares are damaged. The PTM is applied to the inactive traders’ trading in exactly the same way.

Adding the results of the separate PTM analyses together, the MTM estimates that 21,531 shares were damaged. MTMs are often described by a variable called the Relative Trading Frequency. The Relative Trading Frequency is the ratio of the active traders’ trading rate to the inactive traders’ trading rate. The Relative Trading Frequency in our example is equal to 16.

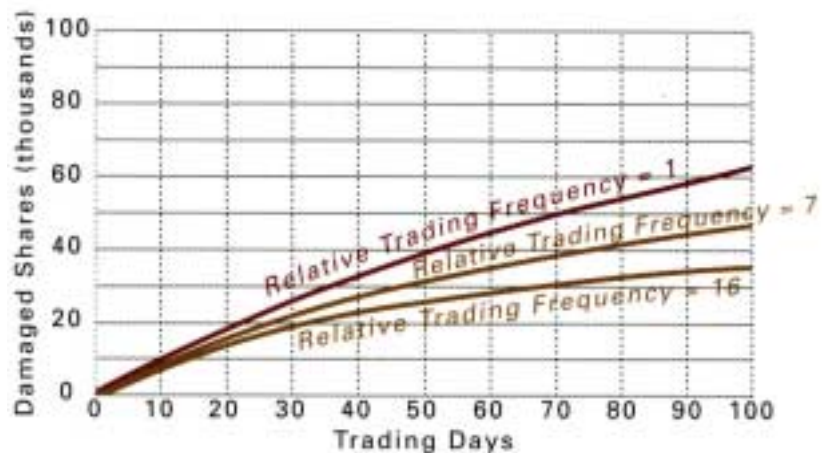
Figure 3 illustrates the MTM’s estimate of the number and timing of damaged share for

Table 2. Multiple Trader Model

High Activity Traders				
Shares available to trade: 20,000/Daily Shares Traded: 8,000				
Day	Untraded Shares Available to Trade	Daily Traded Shares	Damaged Shares	Retraded Shares
	A	B	A*B	(1-A)*B
1	100.0%	8,000	8,000	0
2	60.0%	8,000	4,800	3,200
3	36.0%	8,000	2,880	5,120
	Subtotals	24,000	15,680	8,320

Low Activity Traders				
Shares available to trade: 80,000/Daily Shares Traded: 2,000				
Day	Untraded Shares Available to Trade	Daily Traded Shares	Damaged Shares	Retraded Shares
	A	B	A*B	(1-A)*B
1	100.00%	2,000	2,000	0
2	97.50%	2,000	1,950	50
3	95.05%	2,000	1,901	99
	Subtotals	6,000	5,851	149

Figure 3. Multiple Trader Model Damaged Shares



three Relative Trading Frequency values.

### Accelerated Trader Models

ATMs assume that at any point in time, the probability that a Traded Share will trade is a constant multiple of the probability that an Untraded Share will trade. This multiple is known as Turnover Likelihood Ratio (or Acceleration Factor), Turnover Likelihood Ratios greater than 1 yield more re-trading and fewer damaged shares than the PTM. Turnover Likelihood Ratios less than 1 yield less re-trading and more damaged shares than the PTM.

Table 3. Accelerated Trader Model

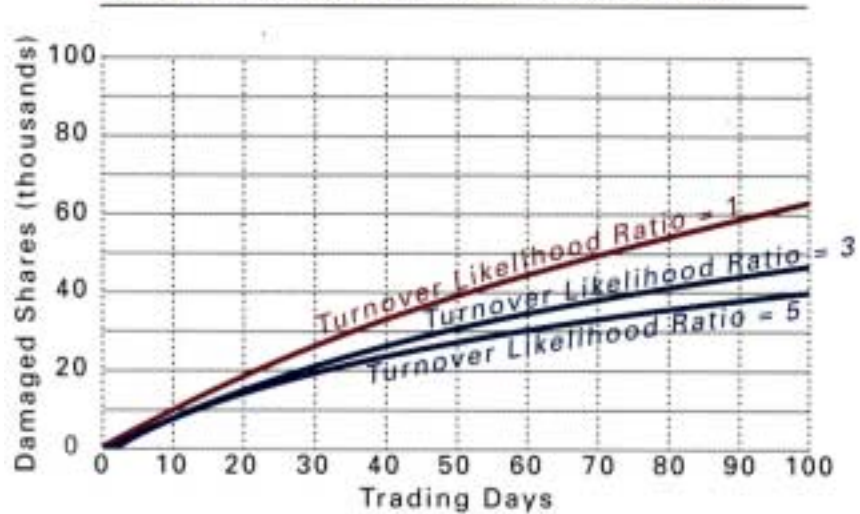
Day	Untraded Shares	Probability of Untraded Shares Trading	Damaged Shares	Retraded Shares
	A	B	C=A*B	10,000-C
1	100,000	10.00%	10,000	0
2	90,000	7.14%	6,426	3,574
3	83,574	6.03%	5,040	4,960
Totals			21,466	8,534

Table 3 presents the ATM analysis of our example class action lawsuit assuming the Turnover Likelihood Ratio is equal to 5.

At the start of the class period all shares available to trade are Untraded Shares and therefore all 10,000 shares traded on the first day are Damaged Shares. On the second day, 90,000 of the shares available to trade are Untraded Shares and the probability that an Untraded Share will trade on the second day is 7.14%. Therefore, on the second day 6,426 of the 90,000 Untraded Shares traded and were damaged. On the third day 83,574 of the shares available to trade are Untraded Shares and the probability that an Untraded Share will trade on the third day is 6.03%.

Therefore on the third day 5,040 shares of the 83,574 Untraded Shares were traded and damaged. An ATM with a Turnover Likelihood Ratio of 5 generates an estimate that 21,466 shares were damaged. Figure 4 illustrates the ATM's estimate of the number and timing of damaged shares using three different Turnover Likelihood Ratios.

Figure 4. Accelerated Trader Model Damaged Shares



### ATMs and MTMs Yield Similar Results

We have presented a highly simplified and stylized discussion of three common stock trading model types. ATMs and MTMs can be adapted to the differing damage calculations appropriate under various sections of the securities laws. ATMs and MTMs generate lower estimates of damaged shares than the PTM by assuming more re-trading.

ATMs and MTMs can yield similar estimates of damaged shares. For example, in Figure





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5 an ATM with a Turnover Likelihood Ratio of 5 and an MTM with a Relative Trading Frequency of 16 yield virtually the same estimate of damaged shares over a 100-day class period.

Though these models can yield similar estimates of the total number of damaged shares, the models differ in their estimates of when shares are first purchased during a class period and thereby damaged. If the inflation varies during the class period, investors who buy and sell during the class period may also be damaged. Thus the choice of trading model may affect damage estimates in class actions in which inflation per share varies over the class period.

### Experts Disagree Over Choice of Trading Model and Parameter Values

Professional disagreement continues over the choice of trading model and parameter values. The sensitivity of damage estimates to model specifications and parameter values can be tested at low cost once the data have been gathered.

### Experts Make Different Use of Data

Even after a model type is chosen and parameter values are assumed, experts **must** decide how best to use available data. For example, experts can make extensive use of public data on insider and institutional trading and holdings or they can make simplifying assumptions. As cases progress experts can use transfer agent and brokerage firm records to refine their analysis.

The use of data and data assumptions can be more important than the choice of model type in explaining differences in damage estimates. For instance, in addition to the parameters that control re-trading, the following three variables must be estimated or assumed:

#### • Market Maker or Specialist Trading

Reported daily volume must be reduced by the amount of market maker or specialist participation to determine the amount of trading by public investors.

#### • Day-Trading

Some public investors sell their purchases the same day they buy stock. These day-traded shares are not usually considered damaged and so public trading volume is further reduced by an estimate of the amount of day-trading.

#### • Shares Available to Trade

Shares available to trade change each day because of changes in the number of shares outstanding, the level of short interest and the levels of insider and institutional holdings.

Figure 5. PTM, ATM and MTM comparison.

